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Mechanical properties of microcantilevers: Influence of the anticlastic effect

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ABSTRACT

Measurements of Young's modulus of microstructures are frequently based on dynamic tests on microbeams. The aim of this work is evaluating if the accuracy of these measurements is affected significantly by the anticlastic effect. A nonlinear model of cantilever's dynamic behavior is thus developed and applied to some characteristic cases. The obtained results show that, even if the introduced nonlinearity is small enough to allow a modal approach to be still applied, the anticlastic effect has a meaningful influence on measurement accuracies as it is evidenced by the dependence of the resonant frequency on vibration amplitudes. The proposed treatise permits determining the appropriate range of excitation amplitudes to be used during the experiments and consequently to reduce appreciably the intervals of uncertainty of the measurements.

1. Introduction

Microcantilevers are widely used nowadays as measurement devices in a broad range of applications: scanning tunneling and atomic force microscopes, micro and nano tribology studies, biology (down to the level of single molecule assay), etc. [1–4].

Material properties of microstructures are also frequently assessed by means of tests on microbeams. Static tests are sometimes performed to correlate the load-to-displacement behavior with the elastic modulus of the material [5,6]. More often, dynamic tests are performed to assess the first bending mode frequency of the studied structure with the aim of evaluating the elastic modulus of the material [7–12]. Since frequency measurements are generally easy to be implemented, the employment of dynamic tests reduces the complexity of the experimental set-up while accuracy is often improved up to the level of a few percent. If the effects of other error sources (geometry, air damping, residual stresses, etc.) are also considered, the intervals of uncertainty in the evaluation of Young's modulus reported in the literature are generally of the order of 10%.

In [13] it has recently been proven that, in the case of slender beams loaded statically by a pure couple, due to the so called anticlastic effect, the flexural behavior of the structure can be significantly affected not only by its geometrical characteristics but also by the entity of the deflections. A slightly deflected microbeam could thus exhibit a different flexural stiffness with respect to the same structure undergoing higher loads. It could be reasonable to assume that the anticlastic phenomenon affects also the dynamic behavior of a cantilever beam. In this case, however, the boundary conditions of the approach given in [13] are not respected, since during each cycle of vibrations the bending moment is not constant but varies along the beam and in time. In the dynamic tests described in the literature, this effect is not taken into account. Only in [12] the influence of the anticlastic curvature is considered, but a static approach is applied. The aim of this work is evaluating the influence of the anticlastic effect on the accuracy of the measurements of the material properties of microstructures.

2. Semi-analytical model

The first modal shape of flexural vibrations of a cantilever beam depicted in Fig. 1 can be described in normalized form as [14]:

$$q_1(\zeta) = \left(\frac{1}{N_2}\right)\{\sin(\beta_1\zeta) - \sinh(\beta_1\zeta) - N_1[\cos(\beta_1\zeta) - \cosh(\beta_1\zeta)]\} \quad (1)$$

where $\zeta = z/L$, with z being the longitudinal coordinate along the beam of length L, while β_1, in the case of a clamped-free beam, is $\beta_1 = 1.875104$, and:

$$N_1 = \frac{\sin\beta_1 + \sinh\beta_1}{\cos\beta_1 + \cosh\beta_1}$$
$$N_2 = (\sin\beta_1 - \sinh\beta_1) - N_1(\cos\beta_1 - \cosh\beta_1) \quad (2)$$

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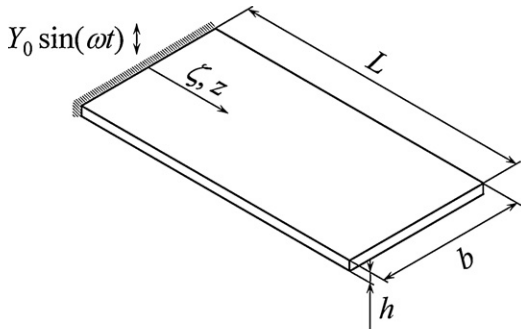

Fig. 1. Microcantilever structure.

The actual shape of the cantilever is thus:

$$u_1(\zeta, t) = \eta_1(t) q_1(\zeta) \tag{3}$$

where $\eta_1(t)$ is the modal coordinate of the first mode. Since in the considered case the eigenfunction expressed by Eq. (1) is normalized in such a way that the maximum value of the displacement is equal to unity, $\eta_1(t) = \bar{y}(t)$ i.e. $\eta_1(t)$ is equal to the amplitude of the displacement of the microbeam.

To take into account the anticlastic effect, the characteristic parameter αb defined by Angeli et al. [13] is:

$$\alpha b = \sqrt[4]{3(1-\nu^2)} \frac{b}{\sqrt{\bar{R}h}} \tag{4}$$

where, referring to Fig. 1, b is the width of the cantilever, h is its thickness, $\bar{R}$ is the curvature radius of the cantilever in the deformed position and ν is the Poisson's ratio of the beam material.

Since the first modal beam curvature $1/R$ is the second derivative of Eq. (1) with respect to ζ:

$$\frac{1}{R(\zeta)} = \left(\frac{\beta_1^2}{L^2 N_2}\right) \{-\sin(\beta_1\zeta) - \sinh(\beta_1\zeta) - N_1[-\cos(\beta_1\zeta) - \cosh(\beta_1\zeta)]\} \tag{5}$$

obviously, according to Eq. (3), the beam curvature at a certain instant of time will be:

$$\frac{1}{\bar{R}(\zeta, t)} = \frac{\eta_1(t)}{R(\zeta)} \tag{6}$$

It is hence possible to adopt the correction factor Φ for the flexural stiffness of the beam as defined in [13]:

$$\phi = \frac{1}{1-\nu^2} - \frac{2\nu^2}{\alpha b(1-\nu^2)} F^*(\alpha b) + \frac{\nu^2}{2\alpha b(1-\nu^2)} f^*(\alpha b) \tag{7}$$

with

$$\begin{aligned} F^*(\alpha b) &= (B_1^* + B_2^*)\sinh\frac{\alpha b}{2}\cos\frac{\alpha b}{2} - (B_1^* - B_2^*)\cosh\frac{\alpha b}{2}\sin\frac{\alpha b}{2} \\ f^*(\alpha b) &= 2(B_1^{*^2} + B_2^{*^2})(\sinh\alpha b + \sin\alpha b) + (B_1^{*^2} - B_2^{*^2} + 2B_1^* B_2^*)\cosh \\ &\alpha b \sin\alpha b + (B_1^{*^2} - B_2^{*^2} - 2B_1^* B_2^*)\sinh\alpha b\cos\alpha b + 2(B_1^{*^2} - B_2^{*^2})\alpha b \end{aligned} \tag{8}$$

and

$$B_1^* = \frac{B_1}{\nu/\sqrt{3(1-\nu^2)}} \quad B_2^* = \frac{B_2}{\nu/\sqrt{3(1-\nu^2)}} \tag{9}$$

$$\begin{aligned} B_1 &= \frac{\nu}{\sqrt{3(1-\nu^2)}} \frac{\sinh(\alpha b/2)\cos(\alpha b/2) - \cosh(\alpha b/2)\sin(\alpha b/2)}{\sinh\alpha b + \sin\alpha b} \\ B_2 &= \frac{\nu}{\sqrt{3(1-\nu^2)}} \frac{\sinh(\alpha b/2)\cos(\alpha b/2) + \cosh(\alpha b/2)\sin(\alpha b/2)}{\sinh\alpha b + \sin\alpha b} \end{aligned} \tag{10}$$

The theory given in Angeli et al. [13] is based on the assumption that the load applied to the beam induces a constant curvature along its length. In the case considered in this work (frequency response of a microcantilever) the curvature varies continuously along the beam. It seems, therefore, reasonable to extend the same approach, thus obtaining a stiffness correction factor Φ that depends on the position ζ along the beam and varies also during each oscillation cycle, i.e. $\Phi = \Phi(\zeta, t)$. What is more, making the hypothesis that the system is slightly nonlinear, i.e. that the correction introduced by taking into account the anticlastic effect influences only slightly the dynamic response of the microbeam, it seems reasonable that a modal approach can still be used. If the usual methodology is applied [14], the "instantaneous" modal stiffness $\bar{K}_1$ of the microbeam can be evaluated as:

$$\bar{K}_1 = \int_0^1 \frac{bh^3 E\phi}{12(R(\zeta))^2} L d\zeta \tag{11}$$

where E is Young's modulus of the beam material.

It must be noted that, with respect to the linear case, the usual expression of the modal stiffness is modified by introducing the correction factor Φ that is integrated along the whole length of the beam. Moreover, due to the mentioned small nonlinearity, the modal stiffness varies during the oscillation cycle, i.e. $\bar{K}_1 = \bar{K}_1(t)$, and therefore it has been indicated as "instantaneous".

The modal stiffness of the linear system K_1 is:

$$K_1 = \int_0^1 \frac{bh^3 E}{12(R(\zeta))^2} L d\zeta \tag{12}$$

Considering Eqs. (11) and (12) and bearing in mind that the limit values of the correction factor Φ are, respectively, 1 and $1/(1-\nu^2)$ [13], it follows that:

$$K_1 \le \bar{K}_1 \le \frac{K_1}{1-\nu^2} \tag{13}$$

In fact, depending on the oscillation amplitude, the "instantaneous" modal stiffness $\bar{K}_1$ could vary slightly between that of a beam-like structure K_1 and that of a plate bent to a cylindrical surface $K_1/(1-\nu^2)$ [15]. These bound values of flexural stiffness refer, respectively, to a plane stress and a plane strain structural model.

The well known expression of modal mass can be used:

$$\bar{M}_1 = \int_0^1 \rho b h [q_1(\zeta)]^2 L d\zeta \approx 0.25 \rho L b h \tag{14}$$

with ρ designating the density of the cantilever.

As it will be shown below, due to the slight variation of $\bar{K}_1$, the response of the system is close to that of a linear system. A resonant frequency, i.e. the frequency at which the normalized frequency response is maximal, can thus still be expressed as:

$$\bar{\omega}_1 = \sqrt{\frac{\bar{K}_{1eq}}{\bar{M}_1}} \tag{15}$$

where $\bar{K}_{1eq}$ is the "equivalent" modal stiffness of the system. Physically, the latter is a kind of time average of the value of $\bar{K}_1$ that cannot be determined analytically, but it has to be obtained numerically (or experimentally).

In the case of excitations due to the harmonic motion of the supporting structure $y_0(t) = Y_0 \sin(\omega t)$, the response of the cantilever is obtained by resorting to a reference frame fixed to the constraint (Fig. 2) [14]. In this case the modal force is given by:

$$\bar{F}_1(t) = a\omega^2 Y_0 \sin(\omega t) \tag{16}$$

where a, in the case of a beam with uniformly distributed mass, is:

$$a = \left(\rho \frac{hbL}{N_2 \beta_1}\right) \{-\cos\beta_1 - \cosh\beta_1 - N_1[\sin\beta_1 - \sinh\beta_1] + 2\} \tag{17}$$

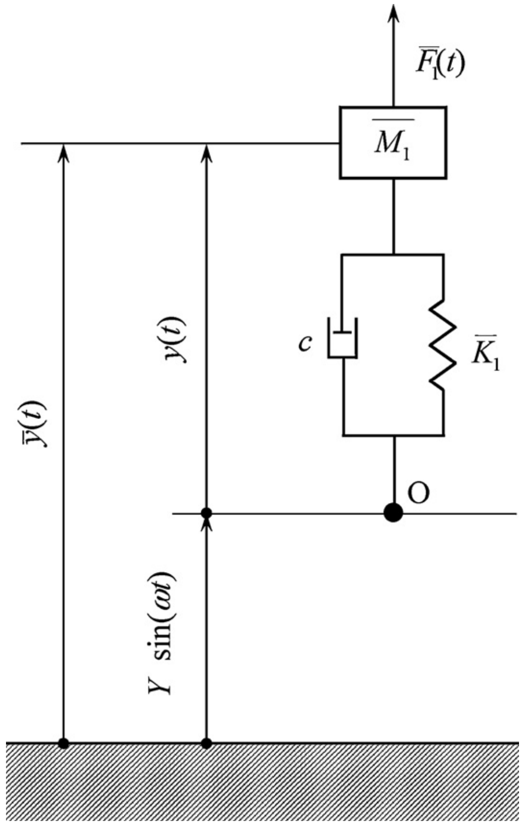

Fig. 2. Scheme of an equivalent one degree of freedom system.

By indicating with c the damping coefficient, the acceleration of the modal mass can hence be computed:

$$\ddot{y}(t) = \frac{1}{\bar{M}_1}(\bar{F}_1 - c\dot{y} - \bar{K}_1 y) \tag{18}$$

The value of the displacement y is thus obtained by integrating twice Eq. (18). The displacement of the modal mass in the absolute reference frame is finally given by:

$$\bar{y}(t) = y + Y_0\ \sin(\omega t) \tag{19}$$

3. Frequency response of the microbeam

The nonlinear beam model described in Section 2 permits the frequency response of the microbeam excited by the motion of the supporting structure to be evaluated. The computation can be restricted to a frequency range close to that of the first natural frequency of the linear beam model ω_1:

$$\omega_1 = \sqrt{\frac{K_1}{\bar{M}_1}} = \frac{h}{2}\left(\frac{\beta_1}{L}\right)^2 \sqrt{\frac{E}{3\rho}} \tag{20}$$

In fact, according to Eq. (13), it is to be expected that the nonlinear frequency response differs only slightly from that of the linear system.

The computational procedure is shown in Fig. 3. For a given excitation amplitude Y_0 of the supporting structure, the frequency is spanned in the interval from $\omega_1 - \Delta\omega$ to $\omega_1 + \Delta\omega$. For each value of the frequency, the actual shape of the beam in a given instant of time is obtained from Eq. (3), while the respective beam shape curvature is evaluated along the beam length using Eq. (6). The flexural stiffness correction factor Φ, due to the anticlastic effect, is evaluated for each instant of time as function of the ζ coordinate by using Eq. (7). The modal stiffness of the beam is then evaluated by introducing this correction factor in the integral given by expression (11). The modal force is hence calculated, allowing the dynamics of the single degree of freedom equivalent system (Eq. (18)) to be solved for a certain instant of time. A new value of beam shape amplitude is thus obtained.

The procedure is repeated in a time loop up to the point when the steady state condition is reached and the final amplitude $\bar{y}_{ss}$ is obtained. The values of $\bar{y}_{ss}$ vs. the vibration frequency are plotted. The frequency $\bar{\omega}_1$, where $\bar{y}_{ss} = \bar{y}_{\max}$ (as pointed out previously, $\bar{\omega}_1$ can still be considered as the resonance frequency), can hence be determined. The whole procedure is reiterated for different values of the excitation amplitude Y_0.

4. Numerical verification

As stated previously, the illustrated semi-analytical model is based on the following simplifying assumptions:

- the nonlinearity of the dynamic behavior of the system is small, thus allowing the modal approach to be still applied;
- the anticlastic effect is evaluated for a beam of non constant curvature.

A nonlinear finite element method (FEM) model, implemented by using 8-node shell elements, is used to verify the validity of these assumptions. Several meshes were tested to optimize the computer time vs. accuracy ratio. A minimum of 6 finite elements along the half-width of the cantilever model is thus used, while a reasonable elements' aspect ratio implied the determination of the number of finite elements along the length of the beam. The geometrical nonlinearities included in this shell FEM model to take into account the anticlastic effect, allow also exploring whether large flexural deflections can induce a meaningful effect on the dynamic response of the microbeam. The model was implemented by using the commercial FEM code Ansys.

A FEM model based on an original 3 node beam element formulation [16] is also developed. The basic idea is that, in each time step of an oscillatory cycle, the shape of the deformed cantilever is used to evaluate the corresponding cross-section stiffness modulus, i.e. the correction factor Φ along the beam. Practically, the "equivalent" modal stiffness $\bar{K}_{1\text{eq}}$ is corrected according to the anticlastic effect model. Obviously, this beam FEM model does not take into account mode shapes different from that of a beam. The model can thus be useful in evaluating quickly and accurately the possible discrep-

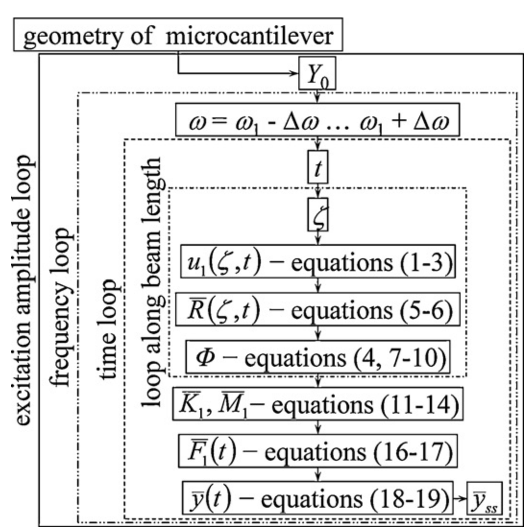

Fig. 3. Flow-chart of the computational procedure.

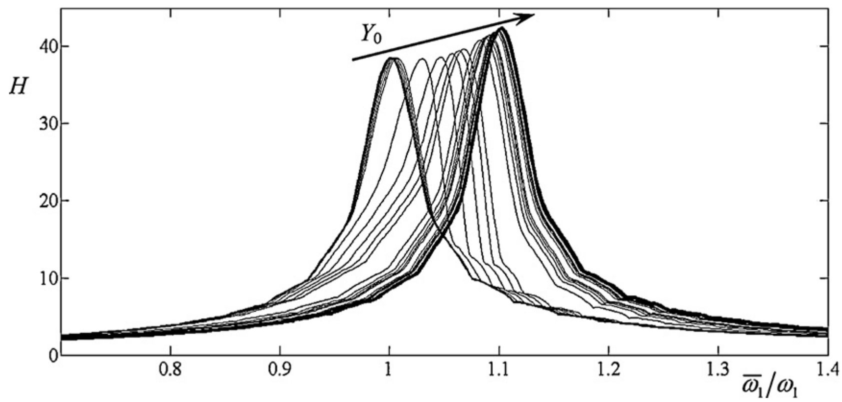

Fig. 4. Normalized frequency response H vs. ratio $\bar{\omega}_1/\omega_1$ for a 100 μm × 100 μm × 3 μm Au cantilever (ν = 0.42).

ancies between the semi-analytical model, which refers to beam theory, and the shell FEM model.

In order to calculate the frequency response of the system, in both FEM models it would have been necessary to make a very large number of simulations for different excitation amplitudes. In fact, a transient simulation until steady state conditions are reached would have been required for each combination of amplitude and frequency. To reduce the computational effort, an alternative approach is proposed in this work. Indeed, the procedures proposed in the literature to determine the material properties of microcantilevers are generally based on measurements of the values of the resonance frequency. The evaluation of the normalized frequency response seems therefore of lesser importance. To attain only the variation of the free vibration frequency, it is thus sufficient to perform a single transient simulation based on the Newmark method [17,18]. The studied beam is hence deflected by an initial tip load. The load is then removed and the free oscillations of the structure are followed in time. By considering subsequent cycles, it is possible to monitor the variation of the frequency of vibrations vs. the respective oscillation amplitudes.

5. Results and discussion

As already pointed out, the evaluation of Young's modulus very often relies on dynamic tests on microstructures. Due to the microfabrication processes, these are frequently characterized by a high width-to-thickness ratio that, in turn, generally induces to consider erroneously that the first modal flexural stiffness is necessarily proportional to the term $1/(1-\nu^2)$ (refer in this regard to Eq. (13)), i.e. to the flexural parameter typical of a plate bent to a cylindrical surface. In Section 2, where the nonlinear model introduced in this work is described, it has been proven that the evaluation of the modal flexural stiffness cannot rely only on the geometry of the considered microstructure, but also on its curvature and vibration amplitude. What is more, an erroneous evaluation of this parameter can imply significant errors, since, as shown by Eq. (13), the "equivalent" modal stiffness $\bar{K}_1$ could vary between that of a beam-like structure K_1 and that of a plate bent to a cylindrical structure $K_1/(1-\nu^2)$. In the case of materials with a high value of Poisson's ratio ν, a significant error in the evaluation of the elastic modulus E could thus be obtained.

Considering then some characteristic cases reported in the literature, the entity of the errors that could result from a non-correct evaluation of the anticlastic effect can be quantified. On the other hand, as stated previously, the proposed semi-analytical model is based on some simplifying assumptions. It follows that a comparison among the results obtained with the proposed theoretical models has to be carried out first.

5.1. Validation of the used models

In Fig. 4 are shown the results obtained by applying the semi-analytical approach proposed in this work to the case of a 100 μm × 100 μm × 3 μm Au cantilever (ν = 0.42) considered in [7]. According to Eq. (20), its first natural frequency is thus $\omega_1 = 4.4 \times 10^5$ rad/s. The cantilever is excited with amplitudes Y_0 in the range from 10 nm to 6 mm, while the assumed damping coefficient is $c = 2.5 \times 10^{-6}$ kg/s corresponding to a damping ratio

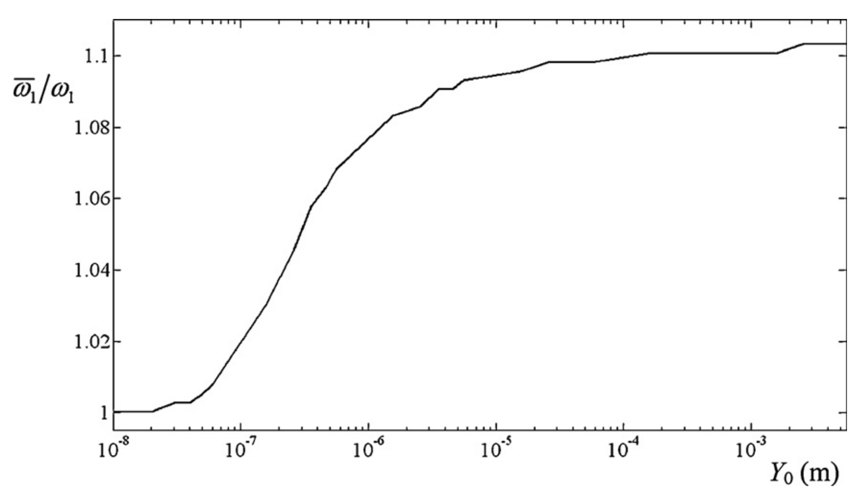

Fig. 5. Dependence of the variation of the ratio $\bar{\omega}_1/\omega_1$ on the excitation amplitudes Y_0 for a 100 μm × 100 μm × 3 μm Au cantilever (ν = 0.42).

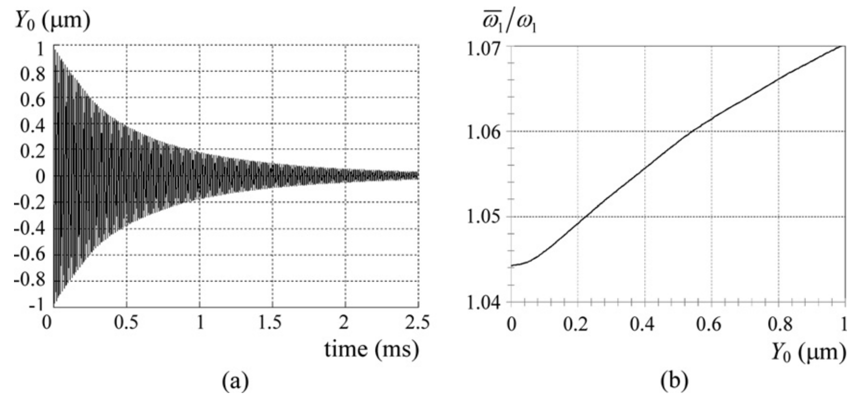

Fig. 6. Shell FEM results: excitation amplitudes in time (a) and variation of the ratio $\bar{\omega}_1/\omega_1$ vs. Y_0 (b) for a 100 μm × 100 μm × 3 μm Au cantilever (ν = 0.42).

of 0.02. It can be observed that, for values of excitation amplitudes lower than or equal to 20 nm, the maximum normalized frequency response $H = \bar{y}_{\max}/Y_0$ occurs for frequencies corresponding to ω_1. On the other hand, for increasing values of excitation amplitudes, the maximum normalized frequency response raises slightly in amplitude while the corresponding frequency increases up to the value of $1.10\omega_1 = \bar{\omega}_{1\max}$. For excitation amplitudes higher than 2 mm, $\bar{\omega}_{1\max}$ remains constant. Obviously, given the length of the microcantilever and the obtained normalized frequency responses, excitation amplitudes higher than about 1 μm (i.e. up to 0.01L) do not have any physical sense, but are considered here for the sole purpose of showing the extremes of the studied behavior, i.e. the maximal theoretical errors on the values of Young's modulus.

These aspects can be enhanced by representing the dependence of $\bar{\omega}_1/\omega_1$ vs. the excitation amplitudes obtained by employing the semi-analytical model on the same cantilever (Fig. 5). It can thus be observed that in the depicted range of amplitudes, the system behaves as a slightly nonlinear hardening system. The results shown in the figure can be explained by considering that, as thoroughly demonstrated in [13], the flexural stiffness of a beam changes depending on the characteristic parameter αb. Eq. (4) shows, in fact, that, for a given geometry and material, this parameter increases for increasing values of the deflection of the microbeam. The flexural stiffness of the beam, and hence $\bar{K}_1$, will thus increase for increasing oscillation amplitudes.

According to Eq. (13), for small vibration amplitudes the dynamic behavior of the cantilever will therefore be that of a beam-like structure. In the case of larger vibration amplitudes, $\bar{\omega}_1$ and thus also $\bar{K}_{1\mathrm{eq}}$, tend to increase up to the point where the behavior of the microstructure will be close to that of a plate bent to a cylindrical surface [13].

A numerical verification, according to the procedure described in Section 4, with a structure having the same material and geometric characteristics as those considered previously in the analytical calculation, was also carried out. In Fig. 6 is shown the free vibration behavior obtained via the shell FEM model: in Fig. 6a are given the amplitudes of the displacements of the microcantilever in time, while in Fig. 6b is depicted the variation of the respective oscillation frequency vs. Y_0. It can be observed that, for amplitudes of the order of Y_0 = 1 μm, the oscillation frequency is 4.7×10^5 rad/s. For longer intervals of time, the tip displacement amplitudes decrease logarithmically, while concurrently the frequency decreases and, for displacement amplitudes smaller than 10 nm, stabilizes at about 4.6×10^5 rad/s.

A comparison of the results obtained by employing the semi-analytical model proposed in this work with those obtained by using the shell FEM model is given in Fig. 7. In order to clarify better the obtained results, in the same figure are presented also the results obtained via the beam FEM model.

Two geometries were considered: a gold 100 μm long microbeam studied in [7] and a 1600 μm long silicon nitride microbeam analyzed in [9]. In the following these two microstructures will be referred to as "stubby" and "slender" beams, respectively. The "stubby" beam is characterized by a length-to-width ratio equal to 1 and a width-to-thickness ratio larger than 30. As it will be discussed below, such a structure can hardly be described by using a beam model. In the case of the "slender" beam, the length-to-width ratio is much higher (i.e. it is equal to 16). From a geometrical point of view, the two considered cases represent, in fact, the upper and lower extremes among the experimental cases reported in the literature and referred to in the following section of the present study.

As it can then be appreciated in Fig. 7, in the case of the "stubby beam" subjected to small excitation amplitudes, a significant difference between the semi-analytical model (as well as the beam FEM model) on the one hand, and the shell FEM model on the other, is observed. This behavior can probably be justified with geometrical considerations. In fact, with respect to a plate model, a beam model seems less suited to describe the dynamics of a thin structure with a length-to-width ratio equal to unity. At small excitation amplitudes, the frequency where the normalized response obtained by employing the shell FEM model is maximal, is therefore already slightly higher than that obtained by using a beam-based

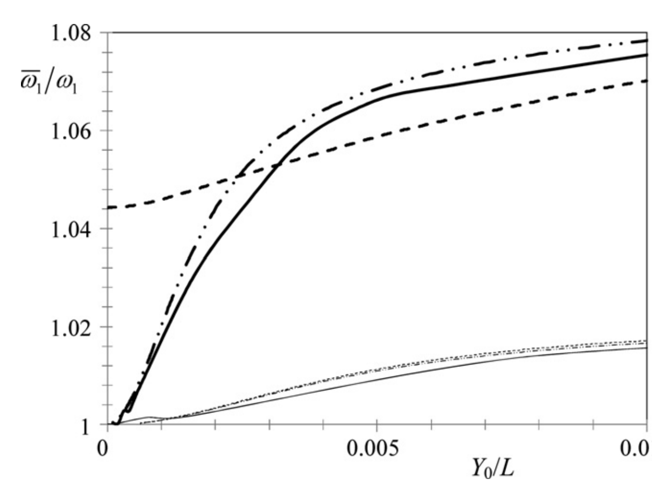

Fig. 7. Variation of the ratio $\bar{\omega}/\omega_1$ vs. excitation amplitudes Y_0 obtained via the semi-analytical (full line), the shell (dotted line) and the beam (dashed line) FEM models for a 100 μm × 100 μm × 3 μm Au cantilever (thicker lines) and a 1600 μm × 100 μm × 0.54 μm silicon nitride (thinner lines) cantilever.

Table 1
Characteristic parameters of microcantilevers considered in the literature and respective uncertainties Δ in the evaluation of E.

Ref.	L (μm)	b (μm)	h (μm)	Material	E (GPa)	ν	Y_{0min} (nm)	Y_{0max} (μm)	Δ_t (%)	Δ_{beam} (%)	Δ_{shell} (%)
[7]	100	100	3	Gold	40	0.42	20	1	16.5	16.4	5.0
[7]	1000	100	3	Gold	40	0.42	2000	10	3.4	4.3	4.4
[9]	100	100	0.54	LPCVD Si_xN_y	95	0.23	3	1	4.9	5.0	4.2
[9]	1600	100	0.54	LPCVD Si_xN_y	95	0.23	250	15	3.3	3.8	3.9
[10]	230	40	7	Nickel	170	0.3	250	2	0.6	0.2	0.3
[8]	380	100	10	Nickel	185	0.3	350	4	1.5	1.6	1.2
[8]	1200	100	10	Nickel	185	0.3	3000	12	0.4	0.2	0.3

approach (semi-analytical and beam FEM). By increasing the excitation amplitudes, all the models show a similar behavior so that, already for an excitation amplitude equal to 0.002L, they give very similar results.

In the "slender beam" case, the behavior is similar for both the semi-analytical and the numerical models, suggesting that a beam approach is correct.

It is, therefore, possible to conclude that, in the case of "slender beams" the semi-analytical model gives reliable results. The case of microstructures with a smaller length-to-width ratio should, however, be avoided unless a thorough numerical analysis is performed. In fact, in the case of small excitation amplitudes, their first natural frequency could differ from that obtained by using a beam-like model, thus limiting the accuracy achievable with a simplified approach.

5.2. Uncertainties of experimental data in the evaluation of Young's moduli

In Table 1 are summarized the uncertainties in the evaluation of Young's moduli, which could result in the case of measurements reported in the literature by using a flexural stiffness model that does not take into account the anticlastic effect. In the last three columns are reported the relative uncertainties in the evaluation of E with respect to a beam in plane stress. Δ_t represents the maximal uncertainty obtained by employing the proposed semi-analytical model with a physically feasible excitation amplitude $Y_{0\,max}$. Δ_{beam} and Δ_{shell} are the uncertainty values obtained, respectively, by using the beam and the shell FEM model. Depending on the structural model adopted for the considered case, the maximal measurement error could be even bigger than the reported uncertainty levels. The reported $Y_{0\,min}$ values represent the minimal excitation amplitudes where the hardening due to the anticlastic effect becomes noticeable.

In [7] the case of gold microcantilevers (ν = 0.42), having dimensions in the ranges h = 3–4 μm, b = 100 μm and L = 100–1000 μm, was considered. The evaluation of the value of Young's modulus E was performed by measuring the first resonance frequency. Given the high width-to-thickness ratio, a plane strain hypothesis was made and thus the term E was substituted with $E/(1-\nu^2)$. However, the values of the vibration amplitudes were not reported although these, as it will be shown below, determine whether the behavior of the microcantilever will approach that of a beam, that of a plate bent to a cylindrical surface or an intermediate value between these two extremes. Considering Fig. 5, it is thus evident that the plane strain hypothesis made in [7] is not realistic, since for physically achievable excitation amplitudes the considered structure will never reach the upper extreme of the curve. The plane strain hypothesis will thus always give rise to estimates of the values of Young's modulus higher than the real ones. Given the fact that the change in the "equivalent" modal stiffness $\bar{K}_{1eq}$, and thus of Young's modulus, is proportional to the square of the change of the frequency at which the normalized frequency response is maximal, a 10% variation of $\bar{\omega}_1$ corresponds to a maximal uncertainty Δ_t = 21% in the determination of E. Limiting, however, the range of excitation amplitudes to maximal values of up to 1 μm (corresponding to a maximal tip displacement of roughly 40 μm), the maximal variation of $\bar{\omega}_1$ will be 8%, corresponding still to a maximal uncertainty in the evaluation of Young's modulus Δ_t = 16.5%.

The uncertainty obtained with the shell FEM model Δ_{shell} is significantly lower than Δ_t (see Table 1). This seems related to the difference of the modal behavior of the structure with respect to that of a beam-like structure. As shown above, a direct consequence of this is also the shift of the free vibration frequency from ω_1 to 1.045ω_1. Such geometries can, therefore, induce an error in the estimation of Young's modulus of the considered material and thus have to be avoided unless an accurate numerical analysis is performed beforehand to evaluate the lower limit of the excitation range where the semi-analytical approach is valid.

When the case of longer microcantilevers (1000 μm × 100 μm × 3 μm) considered in [7] is taken into account, the structure behaves as a beam in a broad range of excitation amplitudes, i.e. up to 2 μm. In this case the maximum uncertainty in the determination of E is therefore significantly lower and both the beam and the shell FEM models produce results comparable to those of the semi-analytical model (Table 1 – see also the above discussion relative to Fig. 7).

Concerning the other cases reported in the literature, the respective results in terms of the uncertainty on the values of Young's modulus, i.e. on the flexural stiffness, are reported in Table 1.

In [9] a study of the behavior of silicon nitride (ν = 0.23) cantilevers is performed making the plane stress assumption, i.e. supposing that flexural stiffness is proportional to E. In this case the authors estimate the measurement uncertainty in the determination of Young's modulus of up to 11%. The influence of the anticlastic effect was not considered even if, according to the data reported in Table 1, it could give rise to an additional uncertainty of up to 5%. As already noted previously, if longer cantilevers are used, the relative influence of this effect decreases. In this case both FEM models are in good agreement with the semi-analytical one. Even for the geometry characterized by a length-to-width ratio equal to one, the geometrical and mechanical characteristics of the used microbeam are such that the dynamics of the structure practically coincides with that obtained via the beam modal model.

In [10] a dynamic approach was used to assess Young's modulus of an electroplated nickel microstructure. Its resonant frequency was measured by the tapping mode of an atomic force microscope (AFM). To correlate the measured frequency with the materials property, a laminar composite beam theory was used where, due to the relatively wide beam with respect to its thickness, $E/(1-\nu^2)$ was employed. This was done again regardless of the entity of excitation amplitudes. A measurement error in the determined value of E was estimated to be 4.1%. Considering the results presented in Table 1, it is evident that in this case the error induced by the anticlastic effect has a limited significance.

In [8] Young's modulus of acoustically actuated electroplated nickel microcantilevers of different lengths, deposited on various substrates with different deposition current densities, was determined. The assumption used to calculate the Young's modulus from the resonant frequency measured via a laser vibrometer, was the plane stress one. Values reported in Table 1 confirm that this approach can be considered appropriate, as it can give rise to uncertainties limited to about 1.5%.

Given the results reported above, it can be concluded that, although due to technological reasons the width-to-thickness ratio of microcantilevers used in the literature to assess Young's modulus of the respective materials is rather large, the adopted correlation between Young's modulus and flexural stiffness seems quite arbitrary. In fact, in some of the cited literature [8,9] the expression for the flexural stiffness is obtained assuming a plane stress behavior, while in other [7,10] the plane strain hypothesis is made. On the other hand, the values reported in Table 1 allow evidencing that the model to be adopted depends on the range of excitation amplitudes. A non negligible error can therefore be made in the estimation of Young's modulus if the oscillation amplitudes and the resulting transition from plane stress to plane strain is not taken into account or eventually when the length-to-width ratio, i.e. the difference in the modal shape from that of a beam-like model, is not considered. In particular, Table 1 shows the upper and lower excitation amplitude bounds where the modal stiffness is that of a beam-like structure or, alternatively, of a plate bent to a cylindrical surface. The upper bound value is, however, generally limited by the condition of having maximum tip displacements significantly smaller than the length of the beam. On the other hand, as evidenced by the shell FEM results, some geometrical configurations could induce slight differences in modal shapes, thus influencing again the equivalent stiffness of the considered structure.

It is to be noted here that a rough estimate of the limiting values of excitation amplitudes can be obtained considering that, in a first approximation, the normalized frequency response is:

$$H = \frac{1}{2\xi} \tag{21}$$

where ξ is the damping ratio. On the other hand, the displacement of the modal mass in the absolute reference frame can be approximated as:

$$\bar{y} = \frac{2h}{3}\left(\frac{L}{b}\right)^2 \frac{(\alpha b)^2}{\sqrt{3(1-\nu^2)}} \tag{22}$$

This expression was obtained considering the deflection of the beam in the middle of its length. The usual Euler–Bernoulli beam model was applied here and consequently the curvature of the beam was taken as directly proportional to $\bar{y}$ and inversely proportional to L^2. Finally, Eq. (4) had to be used since it correlates the curvature and the geometry of the beam to the characteristic parameter αb.

In the case of a beam loaded statically by a pure couple, in [13] it was shown that the cantilever will certainly be in the plane stress (beam-like) state for $\alpha b \leq 0.1$ and in plane strain (plate bent to a cylindrical surface) state for $\alpha b \geq 100$. Eq. (22) allows then determining $\bar{y}$ and thus, via the normalized frequency response of Eq. (21), a rough estimate of the extreme values of Y_0 can be obtained.

When the beam modal model is not applicable, a thorough shell nonlinear FEM analysis has to be performed before planning the experimental assessment of the mechanical properties of the structure.

6. Conclusions

In the case of dynamic tests performed on microcantilevers with the aim of determining the respective material properties, the hardening behavior due to the anticlastic effect has to be considered. In this work a mathematical model is proposed to evaluate the influence of this effect on the dynamics of the cantilever beam. The model is validated via a numerical approach. It is thus applied to some characteristic cases reported in the literature. The obtained results allow establishing that, even if the behavior of the system remains qualitatively similar to the linear modal model, a slight dependence of the "resonance frequency" on vibration amplitudes is present. This nonlinear phenomenon can give rise to significant errors in the evaluation of the stiffness and thus of microstructures' Young's moduli. In the considered cases, it has been shown that this error can have the same order of magnitude as the measurement errors.

The proposed model can therefore be a suitable tool to chose the right experimental set-up and excitation conditions in the dynamic tests. In a first instance, a trial value of Young's modulus E can thus be introduced in Eq. (20) and, according to the procedure given in Fig. 3, a dependence of the type shown in Fig. 5 is obtained for the structure under consideration. This allows establishing the excitation amplitude range to be used in the subsequent experimental tests. During the tests, the excitation amplitudes will then be chosen so that the value of the equivalent stiffness $\bar{K}_{1eq}$ will either be equal to that of a beam-like structure or that of a plate bent to a cylindrical surface. The value of Young's modulus can hence be directly evaluated from Eq. (15).

On the other hand, the obtained results show that, when the geometry of the specimen under consideration is significantly different from that of a slender beam, the proposed semi-analytical beam model might not allow the real behavior of the structure to be accurately predicted. This is due to the fact that the dynamics of the structure could slightly differ from that of a beam. In this case, especially at very small excitation amplitudes, a significant uncertainty could be obtained. For increasing excitation amplitudes, the hardening effect induced by the anticlastic effect seems to dominate and the error becomes negligible. The beam FEM model proposed in this work, based on a nonlinear beam formulation where the stiffness term is corrected according to the theory proposed in [13], can in that case be helpful in achieving a better interpretation of the results.

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