Transversal and axial compliances of optimised flexural hinge shapes

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Abstract

An innovative optimisation procedure allows flexural hinge shapes to be optimised in terms of strength maximisation and parasitic shift minimisation. The analysis of transversal and axial compliances and stress concentration factors of the thus obtained optimised shapes permits then relevant hinge design guidelines to be established.

1 Outline of proposed methodology

A recently developed numerical procedure [1], based on the coupling of parametric optimisation algorithms with automatic FEM meshing and spline function generators, allows a very efficient non-linear optimisation of the shapes of flexural hinges – one of the most frequently used design components in microsytems technologies [2] – Fig. 1. In [1], however, the objective function is defined as the maximisation of the compliance with respect to the sensitive degree of freedom (DOF) φ_z (Fig. 2), while keeping as constraints the strength (stresses smaller than the allowable ones) and the kinematical (eccentricity *e* smaller than a predefined value) conditions of the hinge.

In this work a newly defined objective function is introduced: the integral of the von Mises stresses along the hinge contour, which has to be maximised; the constraint of keeping the stresses smaller than the allowable ones is still kept. The localised effects of the hinge shape modifications are hence taken better into account. This improves considerably the rate of convergence and the robustness of the optimisation process, while making it much more sensitive to the design goals. On the other hand, the eccentricity constraint is replaced by a constraint on the value of the hinge aspect

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ratio $\gamma = L/h_{\min}$ that allows to take better into account the technological limits imposed by the microfabrication processes. In fact, in [1] it was shown that there is a direct correlation between eccentricity and hinge length.

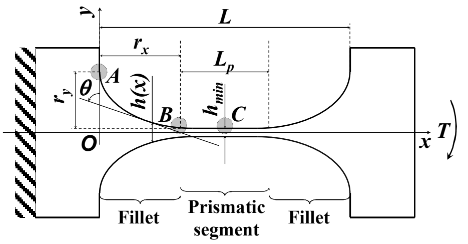

Figure 1: General hinge shape

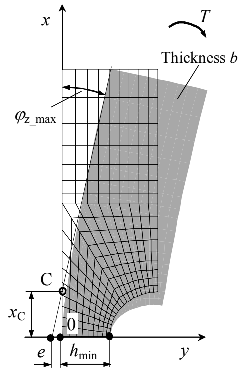

Figure 2: Parameterised shape used in the optimisation procedure

2 Shape optimisation

In a previous work [3] a comparison of the behaviour of several hinge shapes was given for a constant hinge aspect ratio $\gamma = 25$. The following shapes were considered:

- the limit cases of a prismatic beam without stress concentrations (indicated in the following as shape P) and of a conventional right circular (RC) hinge,
- the optimal shapes obtained in classical mechanics for bulky shoulder fillets (parabolic and 'streamline' fillet shapes [4] – based on the authors indicated respectively as the Grodzinski (G), Baud (B) and Thum & Bautz (TB) shape),
- the optimised shapes obtained by using the above optimisation algorithms: a circular optimised hinge with varying prismatic section length (indicated as the optimised circular (OC) shape), the elliptic hinge with $L_p = 0$ (optimised pure elliptical (OPE) shape), the elliptic shape where $r_y = h_{\min}/\pi$ (OEB shape), and a freeform optimised shape (FFO).

Based on the consideration of the compliances and strengths with respect to the sensitive DOF and the respective parasitic shift calculations, it was thus established that, depending on the foreseen application, the FFO shape will be the preferred choice if the main concern is stress minimisation and compliance maximisation, followed in this regard by the 'streamline' (B, TB) and OEB shapes. On the other hand, the OPE and OC shapes provide a good compromise if aiming at a parasitic shift minimisation with still far smaller stresses than those of the RC hinge.

In this work the procedure is extended also to shapes with different values of the hinge aspect ratio. On the thus obtained optimised shapes the comparison of the transversal flexural (out of hinge plane, i.e. around the *y* axis) and axial (along the *x* axis) compliances is performed. In fact, transversal compliance is important in applications where, due to microfabrication technologies, the distance between the suspended moving platform and the respective substrate is limited and insufficient stiffness can lead to sticktion problems. Axial compliances are particularly relevant for slender hinges, as here axial and flexural behaviour can be coupled; for devices loaded axially, this compliance influences also significantly the positioning precision.

3 Transversal and axial compliances

Depending on the hinge aspect ratio γ (only some characteristic values are reported) the obtained optimal shapes can be defined, in terms of shape normalisation characteristics given in [3], as:

Table 1: Geometrical hinge parameters of optimised shapes for different aspect ratios γ

	OC	OPE	OEB
$\gamma = 5$	$r = 1.382\ h_{\min}$	$r_y/r_x = 0.103$	$r_x = 0.18\ L$
$\gamma = 10$	$r = 0.825\ h_{\min}$	$r_y/r_x = 0.062$	$r_x = 0.12\ L$
$\gamma = 25$	$r = 0.59\ h_{\min}$	$r_y/r_x = 0.0314$	$r_x = 0.058\ L$

The FFO fillet shape, remarkably, does not depend on the hinge aspect ratio when γ is > 2.5, which makes this, in a sense, an "absolute optimal shape" [3].

For all the considered aspect ratios γ, the hinge shape was hence shown to have a very small influence on the axial and transversal compliances (always within ± 5% for all but the OPE shape, which is clearly worse than the others).

On the other hand, the influence of the shape on the normalised stresses, i.e. on the stress concentration factors K_σ^*, is significant both in the transversal and in the axial direction. K_σ^* is here defined as $K_\sigma^* = \alpha_{k\_RC}/\alpha_{k\_i}$, where α_{k_RC} and α_{k_i} are the stress concentration factors for the RC and the considered hinge shape respectively.

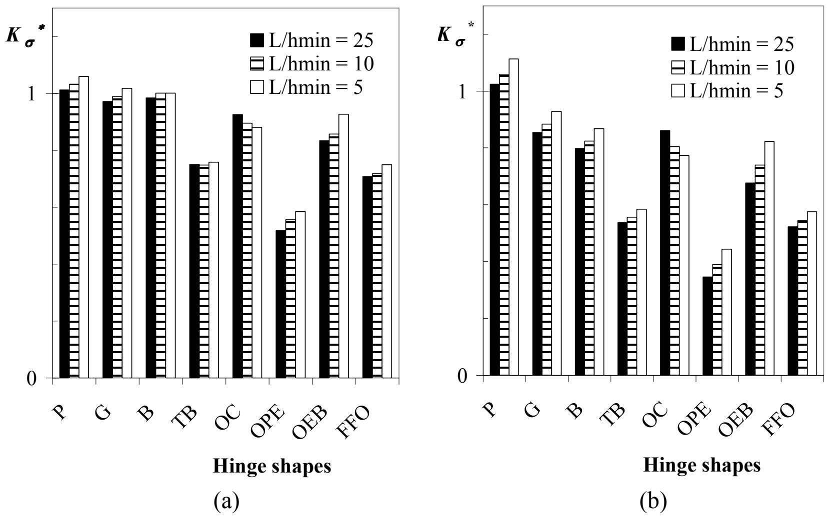

Figure 3: Normalised stresses in the transversal (a) and axial (b) direction

4 Discussion

In terms of decreasing strength for transversal bending, the shapes can be ordered as: B, G, OC, OEB, TB, FFO, OPE. In the axial loading case, this order is: G, B, OC, OEB, TB, FFO, OPE. Thus, the shapes which are most compliant around the primary DOF (FFO and TB) tend to exhibit greater stress concentrations than some of the other shapes. This is particularly relevant since in [3] it was already observed that these intermediate shapes tend also to produce smaller parasitic shifts than the rotationally most compliant ones. Given these considerations, it can thus be concluded that the FFO and TB shapes will be the preferred choice only when the goal is compliance maximisation along the primary hinge DOF, while the G, B, and the optimised OC and OEB shapes will be the favourite choice when both the entity of parasitic shifts with respect to the primary DOF as well as the stress concentration effects in axial and transversal directions are important for the considered application.

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