Modelling and identification of pre-sliding and sliding friction in ultra-high precision positioning systems

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Abstract
Frictional disturbances limit the achievable performances of ultra-high precision positioning devices. Frictional effects are commonly referred to the sliding and pre-sliding motion regimes and can be described via different friction models. The integrated Generalised Maxwell Slip (GMS) model is adopted in this work since it describes reliably frictional phenomena in both regimes via a continuous function.

1. Introduction
Frictional stochastic effects often reduce the performances of ultra-high precision positioning systems. In literature these effects are commonly referred to the sliding and pre-sliding motion regimes, which can be described via different state-of-the-art friction models [1]. The models enable, in fact, to study the behaviour of the considered positioning systems as well as to develop suitable controllers. The integrated Generalised Maxwell Slip (GMS) model is adopted in this work since it describes reliably frictional phenomena via a continuous function [2]. GMS is hence used to describe frictional effects present in a micromanipulation device actuated via a DC motor-gearhead assembly, whose translating elements are supported on linear guideways [3]. The device comprises multiple frictional sources and motion regimes. Elaborated experimental set-ups are therefore used to identify experimentally off-line the respective friction parameters. All these allows developing an overall MATLAB/SIMULINK model of the used positioning device creating the preconditions for compensating efficiently frictional disturbances. The modelled response matches excellently experimental data, confirming thus its validity.

Pre-sliding and sliding friction, friction identification, experimental set-ups, GMS friction model, micromanipulation

2. GMS friction model
The grey-box GMS friction model describes the physics of bodies in relative motion at the surface asperity level, where the state variable \( z \) represents the average deflection of surface asperities. The GMS model enables to describe frictional phenomena in both motion regimes by a continuous function and takes into account all the respective characteristic effects: the Stribeck curve and frictional lag in sliding, the elasto-plastic nonlinear behaviour with non-local memory and hysteresis in pre-sliding as well as the stick-slip effect at the transition between these regimes. The model is based on a parallel connection of \( N \) massless elementary Maxwell-slip blocks with a common input velocity \( v \) (in pre-sliding corresponding to the derivative of \( z \)), and having as output the friction force \( F_i \) of \( i \)-th block (Fig. 1 left). The sliding dynamics of a block is represented by the Stribeck velocity-weakening effect [2]:

\[
s(v) = \text{sgn}(v) \left[ F_s + (F_s - F_c) e^{-|H|v/\delta} \right]
\]

where \( F_c \) is Coulomb friction, \( F_s \) is static friction, \( V_s \) is the Stribeck velocity and \( \delta \) is the shape factor. Two states of either rate-independent hysteresis with non-local memory in pre-sliding or of slip with frictional lag determine hence the behaviour of \( i \)-th block depending on its relative weight \( \alpha \) [4]:

\[
W_i = \alpha \cdot s(v)
\]

The characteristic stiffness \( k \) of each block can be determined by the piecewise approximation of the experimentally obtained data, referring to the pre-sliding behaviour of the system, according to the expressions (cf. Fig. 1 right):

\[
k_i = \Delta F_i / D_i \quad \alpha_i = k_i D_i / F_i
\]

The state of each block will hence be determined based on the following conditions: if \( |F_i(v)| < |W_i(v)| \) the \( i \)-th block sticks:

\[
\frac{df_i}{dt} = k \cdot v
\]

otherwise the \( i \)-th block slips

\[
\frac{df_i}{dt} = \text{sgn}(v) \cdot C \left( \alpha_i - F_i / s(v) \right)
\]

The constant positive number \( C \) is the attraction parameter associated to frictional lag [2]. The total friction force \( F_f \) can thus be calculated as the sum of the forces of all the blocks with the addition of a term related to the viscous friction coefficient \( \sigma \):

\[
F_f = \sum_{i=1}^{N} F_i(t) + \sigma \cdot v(t)
\]

All these expressions can, obviously, be related also to angular velocity \( \omega \) and the respective frictional torques \( M_f \).
3. Experimental identification of friction parameters

The pre-sliding GMS friction parameters of the actuator-gearhead assembly are obtained by incrementally increasing the input voltage to the actuator and simultaneously measuring pre-sliding rotations via a high-resolution incremental encoder, coupled to the actuating assembly via a suitable frame and having an angular resolution of 3.8 μrad. The parameters in the sliding regime are, in turn, obtained by calculating the steady-state angular velocity at the output of the gearhead as the derivative of the readings of the encoder. The frictional torques \( M_f \) in both regimes are calculated by multiplying the motor current with its torque constant \( k_m = 6.6 \text{ mNm/A} \). National instruments real-time equipment is used to control the system and collect and process the acquired data.

The obtained experimental results are characterized by a large time and position variability of up to 15 %, confirming hence the stochastic nature of friction. Based on the average measured data, the parameters of each Maxwell-slip block are obtained according to equation (3). The value of \( C \) is estimated as the inverse value of Stribeck velocity. The friction parameters in the sliding regime are, in turn, obtained by fitting the average experimental data to the shape of Stribeck curve (i.e. equation (1) with the addition of a viscous term) by using the MATLAB optimisation toolbox. The experimental results allow also establishing that there is an asymmetry in frictional behaviour depending on the direction of motion. However, since the resulting difference is smaller than the variability of friction, a single set of nominal parameters can be adopted. The resulting parameters of the GMS friction model are thus reported in table 1.

The frictional behaviour of the linear guideways is, in turn, assessed by tangentially loading the linear slide via a micro-tensile machine with a load resolution of 10 mN, while the resulting displacements are concurrently measured via a laser interferometer. The transmission of the load from the tensile machine to the system is obtained via a carbon-based fibre. Since displacements on the nanometric level are observed even after extended time spans, load increments are made when the stage comes to an almost complete rest. A large number of experiments is hence conducted to assess the variability of friction. The resulting average friction vs. displacement curve, with the respective dispersion (again on the level of \( \pm 15 \% \)), is depicted in Fig. 2. It can thus be seen that pre-sliding is characterised by breakaway forces of up to \( \sim 0.9 \text{ N} \), implying that the frictional effects of the mechanical parts of the system, when reduced to the shaft of the actuator, will be comparable to the variability of friction present in the actuator-gearhead assembly. The biggest frictional contribution in the studied positioning system will hence be that of the actuator. Moreover, due to the reduction ratios, even when the motor-gearhead assembly enters sliding, the mechanical elements will still be in pre-sliding. Most importantly, the value of the pre-sliding displacement of the linear guideways (up to \( \sim 40 \mu \text{m} \)) implies that ultra-high precision positioning will certainly happen when the stage is in pre-sliding. All these considerations allow inferring that only the pre-sliding friction of the linear guideways, whose GMS parameters can be easily obtained from Fig. 2 (cf. equation (1) and Fig. 1 right), is to be considered in the pursuit of nanometric positioning.

The corresponding behaviour of the elements of the considered mechatronics system modelled in MATLAB/SIMULINK allows then confirming that the GMS model describes excellently their actual behaviour (cf. Fig. 2).

### Table 1 GMS model parameters of the actuator-gearhead assembly.

<table>
<thead>
<tr>
<th>Block</th>
<th>( k_f/\text{N} \cdot \text{m}^{-1} )</th>
<th>( \alpha_i )</th>
<th>Sliding regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1159</td>
<td>0.575</td>
<td>( M_f/\mu\text{Nm} )</td>
</tr>
<tr>
<td>2</td>
<td>64.417</td>
<td>0.160</td>
<td>( M_f/\mu\text{Nm} )</td>
</tr>
<tr>
<td>3</td>
<td>13.066</td>
<td>0.059</td>
<td>( \omega_s/\text{s} )</td>
</tr>
<tr>
<td>4</td>
<td>8.027</td>
<td>0.12</td>
<td>( \delta )</td>
</tr>
<tr>
<td>5</td>
<td>2.043</td>
<td>0.051</td>
<td>( \sigma/\text{N} \cdot \text{m} \cdot \text{s} )</td>
</tr>
<tr>
<td>6</td>
<td>0.961</td>
<td>0.034</td>
<td></td>
</tr>
</tbody>
</table>

4. Conclusions and outlook

With the aim of characterising the physical properties related to pre-sliding and sliding friction of a device comprising multiple frictional sources and motion regimes, a translational axis of an ultra-high precision micromanipulation device is used in this work. The parameters of the used GMS friction model are identified off-line separately for the actuator-gearhead assembly and for the linear slide. The obtained results allow confirming the marked stochastic nature of friction in both motion regimes, but also the reliability of the adopted model. The developed procedure of identifying the frictional parameters allows in any case establishing that, due to reduction ratios, the frictional contribution of the actuator is the most significant one. Even when the actuator starts sliding, the downstream elements will still be in pre-sliding, where the ultra-high precision positioning will hence happen. These observations create thus the preconditions for the development of control algorithms based on the adaptive updating of the parameters of the controller so as to efficiently compensate on-line the stochastic variability of frictional disturbances [3]. In future work, more refined control typologies, coupled with identification procedures and metrics suitable to discriminate on-line the influence of the different frictional parameters, will also be considered.

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References