Numerical modelling of piezoelectric vibration energy scavenging bimorphs

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Abstract
A finite element model (FEM) analysis of the coupled electromechanical behaviour of bimorph piezoelectric energy scavengers with varying element types and mesh densities is performed in this work. At first a 3D modal analysis is undertaken to validate the eigenfrequencies and eigenmodes obtained via the uncoupled analytical modal model. A harmonic dynamic analysis is performed next so as to evaluate the voltage output of the system under dynamic loading. Finally, a full transient analysis is done. The validity of the results is enhanced by varying the applied resistive loads.

1 Introduction
Energy scavenging is the process of collecting low level environmental energy and its conversion into electric power. Harvesting of the kinetic vibration energy via bending of piezoelectric bimorphs is particularly advantageous since it is characterised by design simplicity, miniaturization potential and high energy and power densities. In a previous work [1] a thorough coupled modal electromechanical frequency response (FRF) analysis was performed and validated experimentally showing that the applied electrical loads have a nonlinear hardening effect, while the obtainable powers have an intricate dependence on the loads. On the other hand, finite element modelling (FEM) of piezoelectric energy scavenger dynamics has proven to be demanding. Based on recent bibliography, two solutions seem to work best: a complex combination of FEM (for structural simulations) with SPICE® algorithms (for electrical domain simulations), or FEM simulations with separately coupled voltage DOFs for each electrode layer so as to simulate an even charge distribution on piezoelectric surfaces. Although some authors [2] claim that existing FEM piezoelectric capabilities are not suitable to describe the voltage output of vibration
harvesters, in [3] excellent results are achieved with a simple FEM model for unimorph harvesters. The latter methodology is hence adopted in this work, extending it to the bimorph case and to the transient analysis with geometric nonlinearities.

Figure 1: Scavenging bimorph (in this work: \( L = 0.1 \) m, \( b = 20 \) mm, \( h_p = 0.4 \) mm \( h_s = 0.5 \) mm, weight of the tip mass \( M_t = 1 \) g)

2 \hspace{1cm} \textbf{FEM of a piezoelectric energy scavenger}

A typical piezoelectric energy scavenging bimorph configuration (Fig. 1) consists of two piezoceramic layers, one metallic substrate layer, thin copper electrodes (disregarded in the model) and a tip mass. Different mesh densities are employed for the mesh sensitivity analysis. The initial \( G_1 \) density is determined as 30 divisions per beam length, 6 per beam width and 4 per bimorph thickness; \( G_{05} \) is half as dense, while \( G_2 \) is twice denser. After a short investigation of available elements with piezoelectric capabilities in the ANSYS® software, it was determined that brick SOLID226 and tetrahedral SOLID227 elements are suitable for modelling piezo bimorphs. Structural elements SOLID186 and SOLID187 are used to model the substrate and the tip mass. CIRCU94 is the only possible option for modelling electrical resistance. Although the structure and the modelling procedure are quite simple, special attention is to be dedicated to electrical coupling of the piezo layers.

3 \hspace{1cm} \textbf{Modal and harmonic analyses}

The 1\textsuperscript{st} fundamental bending mode is analysed in all considered cases. As it is visible by comparing the uncoupled analytical results obtained by employing the conventional eigenvalue extraction method [4] with the uncoupled ANSYS results, the FEM model is mechanically well conditioned resulting in 1\textsuperscript{st} eigenfrequency errors smaller than 1\% (Table 1). Brick elements are less prone to frequency change with mesh density, while tetrahedral elements show better results with a denser mesh. On the basis of these analyses, a coupled harmonic analysis can be performed in
ANSYS and validated via the coupled electromechanical analytical model proposed in [5]. Two frequency ranges are chosen: 60-70 Hz for a cantilever without a tip mass and 55-65 Hz for that with an $M_t$ tip mass. The excitation acceleration for the coupled analyses is set to 1 m/s$^2$ and a pure resistive load is used. An offset of FEM results in terms of frequencies and voltage levels, limited to roughly 4% with respect to the analytical values, can be observed (Fig. 2a). When the values of the resistive load are varied (10 – 100 kΩ), a shift in the eigenfrequency is visible in both models due to the strong influence of electromechanical coupling (Fig. 2b). FEM results show still slightly different curve shapes with respect to analytical results, probably due to FEM shortcomings in piezoelectric conversion simulations. However, both in terms of eigenfrequencies and maximal voltages, the differences are limited to roughly 1-4%.

Table 1: Modal analysis

<table>
<thead>
<tr>
<th></th>
<th>Frequency [Hz]</th>
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<tbody>
<tr>
<td></td>
<td>Without tip mass</td>
</tr>
<tr>
<td>Analytical</td>
<td>62.5</td>
</tr>
<tr>
<td>Mesh</td>
<td>$G_{05}$</td>
</tr>
<tr>
<td>FEM SOLID226</td>
<td>62.5</td>
</tr>
<tr>
<td>Error</td>
<td>0 %</td>
</tr>
<tr>
<td>FEM SOLID227</td>
<td>62.16</td>
</tr>
<tr>
<td>Error</td>
<td>0.54 %</td>
</tr>
</tbody>
</table>

Figure 2: Harmonic analysis with and w/o tip mass (a) and obtained voltages for varying resistive loads (b)

4 Full transient analysis

A transient analysis, allowing steady state responses in time domain to be captured, is performed next with a sine frequency sweep. Due to long computational times, the
analysis is limited to the close vicinity of the eigenfrequency. SOLID226 elements with \( G_{05} \) mesh are used again, with a load value of 100 k\( \Omega \). At each increment, 16 full excitation cycles are performed in order to get close to steady state. With respect to harmonic runs, transient analysis is better in terms of accuracies of eigenfrequencies (errors < 0.015%) and output voltages (errors < 1%), with the mentioned hardening effect clearly visible. When geometric nonlinearities are included, a decrease of voltage amplitudes is observed (Fig. 3), which can be attributed to large deflections (small cross section rotations during loading) as well as the anticlastic effect.

![Figure 3: Transient analysis with geometric nonlinearities and w/o (a) and with tip mass (b)](image)

5 Conclusions and outlook
A finite element model built in ANSYS was used for modal, harmonic and nonlinear transient analysis. The results were compared with previously established analytical models and good correspondence is achieved, allowing an excellent estimation of maximum obtainable powers.

In future work, the authors will explore different harvester geometries as well as further the influence of mechanical nonlinearities on the bimorph behaviour.

Essential bibliography: