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This Publication has to be referred as:
CHARACTERISATION OF VIBRATION ENERGY HARVESTERS

BLAZEVIC, D[avid] & ZELENIKA, S[asa]*

Abstract: Vibration energy harvesting devices based on piezoelectric bimorphs can be widely applied. In this work the results of an analytical simulation of their behaviour and the respective FEM validation of the obtained results are presented. A preliminary experimental assessment of the performances of commercially available devices is also described. The developed analytical and experimental tools will be used to validate the performances of optimised harvester configurations.

Key words: energy harvesting, piezoelectric materials, modal analysis, experimental assessment

1. INTRODUCTION

Energy harvesting is the process of collecting low level ambient energy and its conversion into electric power. Materials and systems used to perform energy harvesting constitute an important research area with strong industrial potential (Priya & Iman, 2009).

Mechanical vibration energy sources are abundant in most environments and hence show a big harvesting potential in applications such as traffic infrastructure, office buildings, households and manufacturing facilities. A large number of design configurations which explore different principles of harvesting have thus been proposed (electrostatic, piezoelectric, electromagnetics) (Roundy et al., 2005).

Energy harvesting devices based on piezoelectric materials are thoroughly studied due to their high energy density per unit volume, linearity of electromechanical coupling, linear stress-strain relationship in the elastic domain, large strain rates and relatively low resonance frequencies (Roundy et al., 2003).

In order to be able to design optimised energy harvesters, their behaviour in terms of dynamic response (modal shapes and eigenfrequencies), electromechanical coupling and charge distribution, matching with downstream loads, as well as with respect to the experimental assessment of the influence of parameters such as the compliance of the constraints, has to be accurately studied (Roundy, 2005; Sodano et al., 2005).

In this work a modal model of the dynamic behaviour of a cantilever beam loaded at the free end with a proof mass that adapts its first bending eigenfrequency to that of the excitation source is proposed. The study is based on the classical Euler-Bernoulli beam model. The obtained results are validated with those obtained via a Finite Element Model (FEM).

An experimental set-up is developed with the aim of verifying the performances of energy harvesters.

The aim of the work is thus developing the analytical, numerical and experimental tools needed to optimise the behaviour of piezoelectric harvesters in terms of shape and electromechanical coupling so as to maximise the energy conversion efficiency while respecting the strength and other structural constraints.

2. MODAL ANALYSIS OF THE HARVESTER

In order to evaluate the modal shapes and the eigenfrequencies of a cantilever beam with a tip proof mass (Fig. 1), a modal analysis has to be performed.

By considering the force and moment equilibrium, while assuming constant cross sections and masses and neglecting the shear forces acting upon the Euler-Bernoulli beam, the following equation is derived (Genta, 1998):

\[ \rho A \frac{d^2 u}{dt^2} = -EI \frac{\partial^2 u}{\partial z^2} \]  

where \( \rho \) is the material density, \( E \) is its Young’s modulus, \( A \) is the cantilever cross section, \( I \) is its second moment of inertia, while \( u(z, t) \) is beam deflection in the \( z \) direction.

Following the standard modal expansion method

\[ u(z, t) = q(z) \cdot \eta(t) \]  

where \( q(z) \) is the tip deflection while \( \eta(t) \) is a harmonic function equal to \( \eta(t) = \sin(\omega t + \phi) \), the obtained solution is:

\[ EI \frac{d^2 q(z)}{dz^2} = \lambda m q(z) \]  

in which \( \lambda \) denotes the eigenfrequency of the cantilever.

The assumed solution of equation (3) is then:

\[ q(z) = C_1 \sin(\omega z) + C_2 \cos(\omega z) + C_3 \sinh(\omega z) + C_4 \cosh(\omega z) \]

Considering the relevant boundary conditions, and introducing in the conventional model the tip mass, a transcendental equation is derived:

\[ \frac{M}{\rho Al} \left[ \cosh(\alpha l) \sin(\alpha l) - \sinh(\alpha l) \cosh(\alpha l) \right] - \cosh(\alpha l) \cos(\alpha l) = 1 \]  

This equation has to be solved numerically to obtain the coefficients \( \alpha l \). The final dimensionless modal shape is thus:

\[ q(z) = \frac{1}{N} \left[ \sin(\beta z) - \sinh(\beta z) - N \left[ \cos(\beta z) - \cosh(\beta z) \right] \right] \]

with dimensionless coefficients \( \beta, N_1 \) and \( N_2 \) (Genta, 1998).
<table>
<thead>
<tr>
<th>λ1 (Hz)</th>
<th>λ2 (Hz)</th>
<th>λ3 (Hz)</th>
<th>λ4 (Hz)</th>
<th>λ5 (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90.7</td>
<td>1242</td>
<td>3950</td>
<td>8201</td>
<td>13998</td>
</tr>
</tbody>
</table>

Tab. 1. Calculated eigenfrequencies

Fig. 2. Mode shapes of a cantilever beam carrying a tip mass

The model, implemented in MatLab®, was tested using a reference rectangular cantilever. The obtained eigenfrequencies \( \lambda \) are shown in Table 1, while the respective mode shapes are depicted in Fig. 2.

3. FEM VALIDATION OF THE MODEL

The proposed analytical model is validated using the ANSYS® FEM software. Two different types of analysis are performed: that with 2D BEAM3 elements and that with SOLID45 3D elements (Fig. 3). In the FEM models a MASS21 point mass is constrained at the tip of the beam. All degrees of freedom at the clamp are constrained. Both FEM models exhibit a behaviour which validates the results obtained via the proposed analytical model, although the 3D model behaves slightly better, especially at higher eigenfrequencies. When comparing the results of analytical and FEM models, the relative differences are always within 2% (Tab. 2).

4. EXPERIMENTAL ASSESSMENT

In order to validate experimentally the developed models, a suitable experimental set-up is developed and tested by using commercially available piezoelectric harvesters. The set-up consists of a Bruel & Kjær amplifier and exciter controller, a MB Dynamics shaker, Bruel & Kjær accelerometers and a MikroEpsilon laser vibrometer, variable resistive loads and a National Instruments DAQ box. In a preliminary usage of the set-up, two types of commercial energy harvesters produced by MIDE Inc. were used: V21b and V25w. The experiment allowed their behaviour and performance in the dynamic range of \( \pm 3 \) Hz around the first resonant frequency to be determined.

Fig. 3. First fundamental mode shape

<table>
<thead>
<tr>
<th>Res. freq [Hz]</th>
<th>Analytic model</th>
<th>FEM 2D</th>
<th>FEM 3D</th>
<th>Analytic VS 3D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
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<td>91.1</td>
<td>90.6</td>
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<tr>
<td>2.</td>
<td>1242</td>
<td>1237.8</td>
<td>1241</td>
<td>0.99</td>
</tr>
<tr>
<td>3.</td>
<td>3950.2</td>
<td>3740.8</td>
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</tr>
<tr>
<td>4.</td>
<td>8201.1</td>
<td>8291.6</td>
<td>8177</td>
<td>0.99</td>
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<tr>
<td>5.</td>
<td>13998</td>
<td>13781</td>
<td>13931</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Tab. 2. Comparison of analytical and FEM results

Fig. 4. Harvester’s performances vs. tip masses and resistive loads

An acceleration level of 0.633 m/s² was kept constant during the experiments, while each cantilever configuration was tested using 3 different tip masses and numerous resistive loads.

The experiments resulted in expected harvesters’ performances (Fig. 4). The V25w device with larger piezoelectric material volume outputs more power; with comparable tip masses, the resulting power levels are twice as large as those obtained on the V21b device. The output power is proportional to the tip mass. Optimal resistances for the employed loading conditions are in the range of 20-60 kΩ.

The developed experimental set-up is thus suited to characterise the performances of optimized harvester configurations.

5. CONCLUSIONS AND OUTLOOK

In this work it was verified via numerical simulation that the proposed analytical modal model describes excellently the dynamic performances of piezoelectric energy harvesters. On the other hand, the developed experimental set-up proved to be suitable for assessing all the significant parameters of the behaviour of the studied devices.

The tools needed to optimize the output power of piezoelectric harvesters, while taking into due consideration their structural limits, have thus been successfully developed. In the next phase of the work, they will be employed together with optimization algorithms and electro-mechanical coupling relations to upgrade the performances of available harvesters and design a new class of optimized devices.

6. REFERENCES


