

# Analytical characterization and experimental validation of performances of piezoelectric vibration energy scavengers

E. Brusa<sup>a</sup>, S. Zelenika<sup>\*b,c&</sup>, L. Moro<sup>b&</sup>, D. Benasciutti<sup>b&</sup>

<sup>a</sup>Politecnico di Torino – Dept. Mechanics, Corso Duca degli Abruzzi 24, 10129 Torino, Italy

<sup>b</sup>University of Udine, DIEGM, Tech-UP Laboratory, Via delle Scienze 208, 33100 Udine, Italy

<sup>c</sup>University of Rijeka – Faculty Eng., Dept. Mech. Eng. Des., Vukovarska 58, 51000 Rijeka, Croatia

## ABSTRACT

One of the main requirements in wireless sensor operation is the availability of autonomous power sources sufficiently compact to be embedded in the same housing and, when the application involves living people, wearable. A possible technological solution satisfying these needs is energy harvesting from the environment. Vibration energy scavenging is one of the most studied approaches in this frame. In this work the conversion of kinetic into electric energy via piezoelectric coupling in resonant beams is studied. Various design approaches are analyzed and relevant parameters are identified. Numerical methods are applied to stress and strain analyses as well as to evaluate the voltage and charge generated by electromechanical coupling. The aim of the work is increasing the specific power generated per unit of scavenger volume by optimizing its shape. Besides the conventional rectangular geometry proposed in literature, two trapezoidal shapes, namely the direct and the reversed trapezoidal configuration, are analyzed. They are modeled to predict their dynamic behavior and energy conversion performance. Analytical and FEM models are compared and resulting figures of merit are drawn. Results of a preliminary experimental validation are also given. A systematic validation of characteristic specimens via an experimental campaign is ongoing.

**Keywords:** energy scavenging, shape optimization, electromechanical coupling, structural mechatronics, piezoelectric materials, experimental validation

## 1. INTRODUCTION

Pervasive technologies are one of the most challenging innovations in mechatronics systems aimed at providing distributed or intelligent monitoring of the environment, of structures and of people. These are often ubiquitous distributed low-power wireless sensors being used, for instance, to monitor mechanical and physical properties as well as damage in structures, lowering the costs of surveillance and monitoring operations. A marked development of miniaturized sensors and controllers was performed to implement pervasive technologies in some critical environments. Continuous monitoring of all functional parameters relevant to the prevention of catastrophic events, the interruption of vital services or, in the case of people, to the advent of traumatic states and health risks, is hence now possible<sup>[1-3]</sup>.

One of the key features of the design of sensor networks is the possibility to supply the needed power over a large space, although the power requirement of each sensor may be very low. Unfortunately, currently available batteries do not comply with the requirements of autonomy and system miniaturization or wearability. A valid solution was found in energy conversion of the power available in the same environment where the sensors are located<sup>[3-5]</sup>.

The so-called collocation of the power supply and the sensor decreases energy losses in power transmission, assures a real time on-demand supply and very often allows reducing the dimensions of the used device. Sometimes collocation is required by the functionality of the smart system. For instance, elder care technology is based on the transmission of alarm signals and information about some vital biometric parameters of the monitored patients to medical personnel

\* sasa.zelenika@riteh.hr; phone: + 385 – (0)51 – 651538; fax: + 385 – (0)51 – 651416; www.riteh.hr

& Work supported by the “*TECH-UP*” project for the development of pervasive technologies of the Friuli Venezia Giulia Region, Italy, the “*Ultra-high precision compliant devices for micro and nanotechnology applications*” project of the Croatian Ministry of Science, Education and Sports and the “*Theoretical and Experimental Analysis of Compliant Mechanisms for Micro- and Nanomechanical Applications*” project of the Italian Ministry of University and Research.

within the same building, e.g. the hospital. In this case the sensor is part of a wearable system, which transmits measurement data, such as acceleration, temperature or blood pressure, to a central archive. Local encoding, data elaboration and transmission via radio frequency systems are all typical functions of the wearable package. The sensor, the processor and the communications unit all rely on localized power generators, which must assure sufficient autonomy within the volume and weight constraints imposed by wearability. Miniaturized energy conversion systems are proposed to satisfy these requirements<sup>[1, 2, 6]</sup>.

Energy harvesting of vibrations has recently gained significant attention<sup>[2, 7, 8]</sup>. In practice, power is generated in situ by converting kinetic energy of vibrating resonant structures into energy storage of electric charge. Power generation can be performed via several approaches. Among the proposed solutions, piezoelectric conversion is useful because of its inherent electromechanical coupling between mechanical strain and electric field. It allows an easy microfabrication in case of micro-electro-mechanical systems (MEMS) and assures good performances. These can be achieved, for instance, by cantilevers made of a metallic substrate onto which piezoelectric material is surface bonded. The structures usually vibrate at their first resonance frequency<sup>[9, 10]</sup>, while vibration is imposed by external environment or by people. Energy is hence scavenged instead of being lost in the environment. These aspects motivate a study of optimized piezoelectric vibration scavenging devices. A complete list of the relevant specifications of such devices and of the related design parameters was not found in literature, although important contributions were identified<sup>[2, 11-14]</sup>.

The design of piezoelectric vibration energy harvesting devices is hence dealt with in this work with the aim of formulating suitable optimization criteria. This is performed by means of a coupled electromechanical model taking into account all the main design parameters. Two trapezoidal optimized geometries are therefore investigated and compared to the conventional rectangular energy scavenger. Besides implementing already validated analytical approaches<sup>[11, 12]</sup>, the performances of the scavengers are evaluated also by means of a numerical approach based on the finite element method (FEM). In fact, the detailed design of the structure implies the need of a thorough analysis of the stress, strain and charge distributions on the piezoelectric transducer. These aspects can be poorly predicted by analytical methods, since these are generally mainly focused on the evaluation of the average power output of the transducer.

The main goal of defining a simple mechatronics approach to the design of the energy scavenger is hence achieved. Moreover, the work allowed a preliminary set-up of an experimental apparatus for the validation of the proposed numerical approaches. It could thus be proven that the proposed optimized layouts provide improvements of energy harvesting performances. A thorough experimental campaign is hence being carried on.

## 2. ANALYTICAL MODELS OF PIEZOELECTRIC VIBRATION ENERGY SCAVENGERS

In designing energy harvesters, the main goal is assuring the highest possible energy conversion within the limits imposed by the energy storage system and the volume constraints. Design parameters include thus material selection, geometry definition as well as the effective modeling of the electromechanical coupling.

A reference layout of the considered energy scavengers is depicted in Fig. 1. The cantilever is dynamically excited at the fixed end at a given frequency and loaded at the free end with a proof mass which magnifies the vibration amplitudes and adjusts the resonant frequency of the system to that of the excitation thus maximizing transmissibility. Piezoelectric layers (indicated in Fig. 1 with 1) are surface bonded to the metallic shim beam structure (indicated with 2). The strain of the piezoelectric material converts, via electromechanical coupling, vibration energy into an electric charge distribution inducing an electric field between the upper and the lower electrodes. The field is transferred via an electric circuit that can include either an energy storage device, such as a capacitor, or a resistive load where harvested power is dissipated.

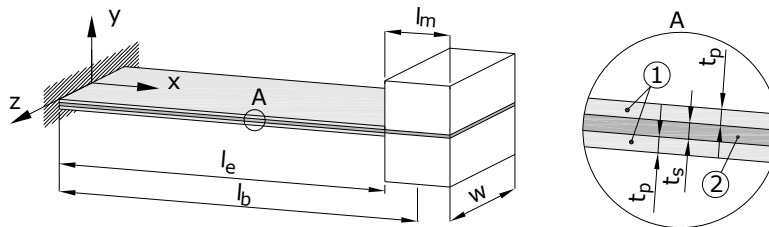


Fig. 1. Basic layout of a piezoelectric vibration energy scavenger.

In the considered configuration, flexural excitation is preferred since several authors demonstrate that this choice assures the best energy conversion efficiency<sup>[2, 11]</sup>. In this work it is verified (see below) that the value of the resonance

frequency for the first flexural mode is far enough from the other eigenfrequencies and especially the torsional modes which sometimes can be close to the first one. Since external excitation is only approximately predictable, an eventual necessity to cover a determined frequency bandwidth can then be addressed either by employing an assembly of energy scavengers with close resonance frequencies<sup>[14]</sup> or by applying a passive<sup>[13]</sup> or an active<sup>[15]</sup> tuning system.

## 2.1 Roundy's model

A reference structure widely analyzed in literature is the rectangular scavenger shown in Fig. 1. An analytical model which describes the behavior of the scavenger at its first flexural resonance frequency<sup>[9, 10]</sup> was developed by Roundy and Wright<sup>[11]</sup>. The bimorph beam structure is approximated therein by an equivalent homogenous solid with an effective moment of inertia  $I$ :

$$I = 2 \left[ \frac{wt_p^3}{12} + wt_p b^2 \right] + \eta \frac{wt_s^3}{12} \quad (1)$$

where the parameter  $b$  is  $b = (t_s + t_p)/2$ , while the ratio of elastic moduli is  $\eta = Y_s/Y_p$  (index  $s$  indicates the metallic shim,  $p$  the piezoelectric material). Indicating with  $l_b = l_e + l_m/2$  (see Fig. 1), the resulting maximum bending stress in a cross section of the scavenger is thus:

$$\sigma(x) = \frac{M(x)b}{I} = \frac{F(l_b - x)b}{2 \left[ \frac{wt_p^3}{12} + wt_p b^2 \right] + \eta \frac{wt_s^3}{12}} \quad (2)$$

In eqs. (1) and (2)  $x$  is the beam longitudinal coordinate,  $M$  is the bending moment due to the inertial load of the proof mass, while the stress is computed at a distance  $b$  from the neutral axis of the cantilever. An average stress along the piezoelectric layer is then computed to determine the resulting power generation.

For a lumped parameter model, the resonance frequency is:

$$f_n = \frac{1}{2\pi} \sqrt{\frac{Y_p}{mk_1 k_2}} \quad (3)$$

where  $k_1$  and  $k_2$  are parameters characterized by the geometry of the scavenger. For a rectangular structure they are:

$$k_1 = \frac{6b(2l_b - l_e)}{(2t_p^3 + 24t_p b^2 + \eta t_s^3)w}, \quad k_2 = \frac{l_b^3}{3b(2l_b - l_e)} \quad (4)$$

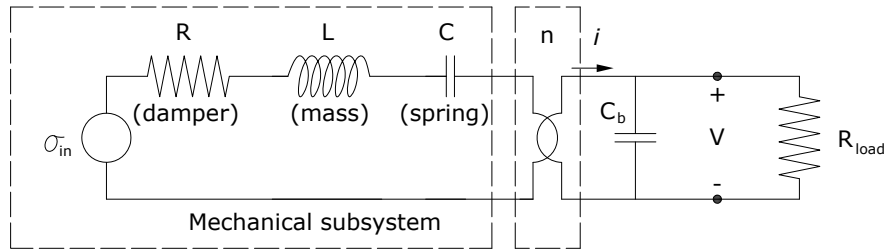


Fig. 2. Electromechanical analogy applied to the model of the piezoelectric energy scavenger.

Electromechanical coupling can be approached via the electromechanical analogy<sup>[11, 16]</sup>; the equivalent circuit of the device is hence represented in Fig. 2. Mechanical motion of the energy scavenger is here represented by the equivalent electric circuit upstream of the primary winding of the transformer where each of scavenger's properties is replaced by the equivalent electric component. The transducer dynamics can thus be described as:

$$\sigma_{in} = L\ddot{S} + R\dot{S} + \frac{S}{C} + nV, \quad i = C_b \dot{V} + \frac{V}{R_{load}} \quad (5)$$

where the voltage generator represents the equivalent stress  $\sigma_{in}$  due to vibrations.  $L$ ,  $C$  and  $R$  designate respectively the equivalent mechanical inertia, compliance and damping of the bimorph, while  $S$  represents mechanical strain. Electromechanical coupling is represented by the transformer with the equivalent turns ratio  $n$ . At the secondary winding

end of the model,  $C_b$  indicates piezoelectric capacitance and  $R_{load}$  the external resistive load. The condition of having a pure resistive load connected to the electrodes is not necessarily the most realistic one, since often the electric load consists of rechargeable batteries and other capacitive loads. However, it is simple and useful not only for estimating the resulting power, but also for giving the designer more intuition about the system<sup>[11]</sup>.

Electromechanical coupling relates voltage  $V$  to mechanical stress  $\sigma$  via the piezoelectric coefficient  $d_{31}$ :

$$\sigma = -d_{31} \frac{aY_p}{2t_p} V \quad (6)$$

where  $a$  is equal either to 1 or 2 for piezoelectric layers connected respectively in series or in parallel<sup>[11]</sup>.

When the excitation frequency  $\omega$  matches the first resonant frequency of the scavenger, the produced voltage  $V$  will assume the value<sup>[11]</sup>:

$$V = \frac{A_{in}}{k_2} \frac{j\omega \frac{2Y_p d_{31} t_c}{a \varepsilon_s}}{j\omega \left( k_{31}^2 \omega^2 + \frac{2\zeta \omega}{R_{load} C_b} \right) - 2\zeta \omega^3} \quad (7)$$

where  $A_{in}$  is the amplitude of the excitation acceleration,  $\zeta$  is the mechanical damping ratio<sup>[10]</sup>,  $\varepsilon_s$  is the static permittivity of the piezoelectric material, while  $j$  is the imaginary number. It is worth noting here that both the parameter  $k_2$  and the piezoelectric capacitance  $C_b$  depend on the scavenger shape. For the conventional rectangular scavenger of Fig. 1,  $k_2$  is given by eq. (4), while the capacitance will be:

$$C_b = \frac{\varepsilon_s w l_e}{2t_p} \quad (8)$$

In the same resonance conditions, the average power dissipated on the resistor  $R_{load}$  will hence be:

$$P = \frac{|V|^2}{2R_{load}} = \frac{A_{in}^2 R_{load} C_b^2 \left( \frac{2Y_p d_{31} t_p}{a \varepsilon_s k_2} \right)^2}{2\omega^2 [(k_{31}^4 + 4\zeta^2)(R_{load} C_b \omega)^2 + 4\zeta^2 k_{31}^2 R_{load} C_b \omega + 4\zeta^2]} \quad (9)$$

Although this model is simple, it allows the key parameters of electromechanical coupling to be appreciated. The coupling coefficient  $k_{31}$  is the relevant figure of merit for the efficiency of electromechanical coupling, while capacitance  $C_b$  is the key electric parameter for circuitry design.

The maximal average power is obtained when the resistor  $R_{load}$  assumes the value  $R_{load\_opt}$ :

$$R_{load\_opt} = \frac{2\zeta}{\omega C_b \sqrt{4\zeta^2 + k_{31}^4}} \quad (10)$$

## 2.2 Guyomar's model

In the analytical model developed by Guyomar and coworkers<sup>[12]</sup>, the bimorph scavenger with proof mass  $M$  at its free end is represented as an equivalent mass-piezo-spring-damper system such as depicted in Fig. 3. With  $K_s$  is indicated the stiffness of the scavenger and with  $c$  its damping coefficient. External excitation given by force  $F$  results in a displacement  $u$  of mass  $M$  and is opposed by the restoring force  $F_p$  of the piezoelectric element, that of the equivalent spring as well as the viscous force of the damper. The respective constitutive equations can thus be written as:

$$F_p = K_{pE} u + \alpha V, \quad I = \alpha \dot{u} - C_b \dot{V} \quad (11)$$

where  $K_{pE}$  is the stiffness of the piezoelectric element,  $\alpha$  is the so-called force factor and  $C_b$  is the piezoelectric capacitance.  $I$  and  $V$  indicate respectively the outgoing current and voltage on the piezoelectric element. Considering the dynamic equilibrium of such a system, the voltage across the piezoelectric element will thus be:

$$V = \frac{\alpha R_{load}}{1 + j\omega C_b R_{load}} j\omega u_m \quad (12)$$

where  $u_m$  is the amplitude of displacement  $u$ . The other used symbols have the same meaning as in the case of Roundy's model. The average power dissipated across the resistor  $R_{load}$  will in this case be given by:

$$P = \frac{|V|^2}{2R_{load}} = \frac{R_{load} \alpha^2}{1 + (\omega C_b R_{load})^2} \frac{\omega^2 u_m^2}{2} \quad (13)$$

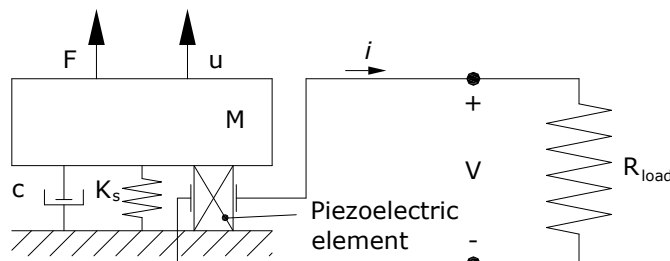


Fig. 3. Electromechanical model of the piezoelectric vibration energy scavenger.

In this case the average output power can therefore be determined knowing the electrical parameters of the system and measuring the amplitude of the motion of the proof mass, i.e. only when experimental or numeric data on the entity of this displacement are available. Eq. (13) will have a maximum when the resistor  $R_{load}$  will have the value:

$$R_{load\_opt} = \frac{1}{\omega C_b} \quad (14)$$

which is similar to eq. (10) provided by Roundy's model.

### 3. OPTIMIZATION OF SCAVENGER SHAPES

The position and the area covered by the piezoelectric layers on the scavenger are important design parameters since electromechanical coupling depends on electric capacitance which is, in turn, dependent on the area and the shape of the transducer itself. In literature it is thus suggested to cover the largest possible area to maximize energy conversion<sup>[17]</sup>, but this criterion does not seem compatible with active vibration control theory<sup>[18]</sup>. In fact, the highest conversion can be achieved if the piezoelectric material is strained in every portion of the layer close to its strength limits. If the surface of the cantilever is covered by the piezoelectric material regardless of the strain state, overall performance can be limited. Moreover, since the surface of the electrode is integer, i.e. the charge resulting from mechanical strain anywhere on the layer will be uniformly distributed on it, a sort of averaging effect is obtained. The design goal is therefore that of shaping the surface of the piezoelectric layer so that, for a vibration mode chosen for energy scavenging, the piezoelectric material is located as much as possible in correspondence of strain concentration on beam's surface. Each mode implies hence a different optimized scavenger shape, thus making difficult the design of the piezoelectric geometry when several modes are excited. However, when the main aim is making use of the first bending resonance of the cantilever, as is the case considered in this work, the criterion of uniform strength, conventionally used in wheels and springs technologies, seems appropriate, since it allows to store within the material the highest possible elastic potential energy<sup>[9, 10]</sup>. For a given load condition, the geometry of the structure is therefore herein designed to approximate as much as possible a constant stress state all over the beam.

In the conventional rectangular scavenger, the bending moment and the resulting normal stress linearly decrease from the fixed to the free end. On the other hand, a triangular beam shape allows obtaining a uniform stress distribution along the cantilever surface and correspondingly on the bonded piezoelectric layers. Given the necessary place for the fixation of the proof mass and other technological constraints, this optimized shape is approximated by a trapezoidal one referred to in the following as "trapezoidal shape". An interesting alternative could be represented by clamping the minor basis of the trapezoid instead of the large one, i.e. using a "reversed trapezoidal shape"; this increases significantly the stress at the fixation, while the larger area at the free end facilitates the positioning of the proof mass but it will be weakly stressed. According to the outlined criteria, the trapezoidal and the reversed trapezoidal shapes are given in Fig. 4.

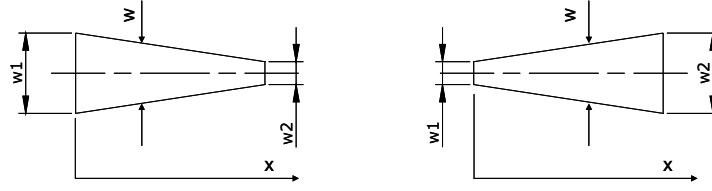


Fig. 4. “Trapezoidal” (left) and “reversed trapezoidal” (right) scavenger shapes clamped along the side of width  $w_1$  and with the proof mass (not shown for clarity reasons) affixed at the free end of width  $w_2$ .

These two devices have a variable width  $w(x)$  along the beam. For a given cross section location  $x$ , the corresponding width, as function of beam length  $l_b$ , for the trapezoidal and the reversed trapezoidal shape, will respectively be:

$$w(x) = \frac{(w_1 - w_2)(l_b - x)}{l_b} + w_2, \quad w(x) = \frac{(w_2 - w_1)x}{l_b} + w_1 \quad (15)$$

Stresses and their distributions along the scavenger length for a given proof mass were computed for the trapezoidal shape of decreasing  $w_2$  and constant  $w_1$ , while these bounds were inverted in the case of the reversed trapezoidal shape. The obtained results are compared in Fig. 5 in terms of the dimensionless stress (with  $F$  being the inertial load due to the proof mass, while the cantilever dimensions are referred to Fig. 1) as function of  $x$ .

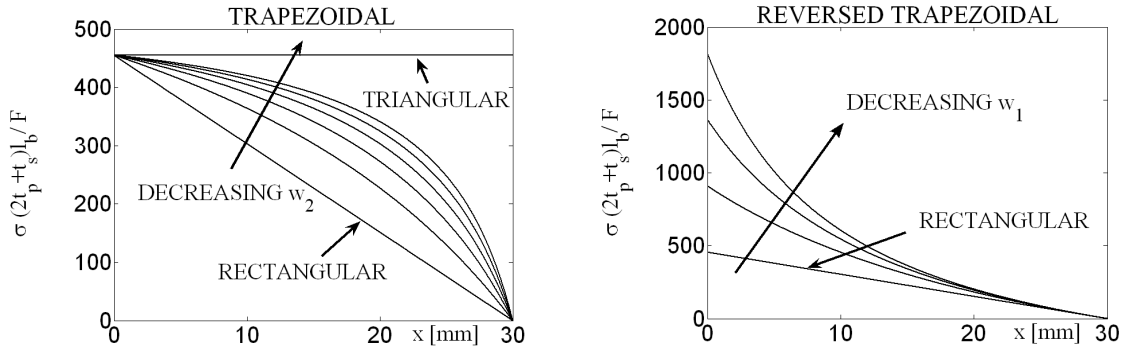


Fig. 5. Stress distribution along the scavenger length computed for the proposed layouts for a constant inertial force.

It can be noted that the reversed layout provides a fairly large stress at the fixation, although this is concentrated and decreases very fast along the beam length. The highest values may even be above the ultimate strength of the materials and sometimes critical for the fatigue life of the device. The trapezoidal shape provides, in turn, a more uniform distribution with the maximum stress much lower than in the previous case.

### 3.1 Optimized shapes

A rectangular shaped energy scavenger replicating the typical geometry proposed in Fig. 1 is taken as reference for the study. It is constituted by a metallic structure made of stainless steel, while the considered piezoelectric material is PSI-5A4E lead-zirconate-titanate (PZT) commercialized by Piezo Systems Inc.<sup>[19]</sup> The piezoelectric layers are supposed poled for series operation since this provides the highest voltage between the upper and lower electrodes. The respective material characteristics used in this work are given in Table 1.

Table 1. Main material characteristics of the reference structure.

Parameter	Value
Density of piezoelectric material	$\rho_p = 7.8 \text{ g/cm}^3$
Young's modulus of piezoelectric material	$Y_p = 6.6 \cdot 10^4 \text{ MPa}$
Young's modulus of steel	$Y_s = 2.06 \cdot 10^5 \text{ MPa}$
Electromechanical coupling coefficient	$k_{31} = 0.14$
Permittivity of free space	$\epsilon_0 = 8.8542 \cdot 10^{-12} \text{ F/m}$
Static permittivity	$\epsilon_s = \epsilon_r \cdot \epsilon_0 = 1258 \epsilon_0 \text{ F/m}$

According to the above analysis, geometric requirements on scavenger shapes include the values of piezoelectric layers' surface area and maximum width. To investigate the role of these two parameters, optimized geometries having either the same maximal width or the same total volume of the piezoelectric material are compared. In a first instance, a maximal transversal dimension of a determined value  $w$  is thus assumed for the rectangular, trapezoidal and reverse trapezoidal shapes. In the second case the widths  $w_1$  and  $w_2$  of both the trapezoidal configurations are varied while keeping constant the volume of all the three considered scavenger shapes. Since the capacitance of the piezoelectric transducer depends on its surface, the two approaches give different results in terms of electromechanical coupling.

In Table 2 and Fig. 6 are given the geometric characteristics of the prototypes obtained from the analysis while keeping the first flexural eigenfrequency of the structure, described for the rectangular scavenger by eq. (3), constant for all the considered geometric configurations. A typical value of 50 Hz, corresponding to the vibrations induced by sources in the domestic and industrial environments, is selected, while the inertial load is constituted by a cubic mass of 10 grams.

Table 2. Geometric parameters of the considered scavenger shapes.

Parameter	Value
Length of proof mass	$l_m = 10$ mm
Length of electrode for rectangular shape	$l_e = 25$ mm
Width of beam for rectangular shape	$w = 14$ mm
Thickness of metallic layer	$t_s = 0,1$ mm
Thickness of piezoelectric layer	$t_p = 0,2$ mm

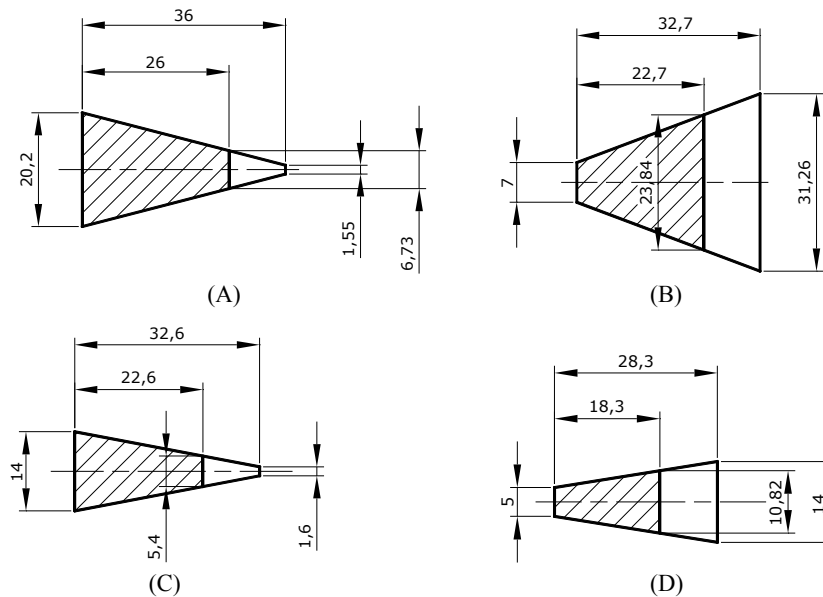


Fig. 6. Optimized energy scavenger prototypes having equal resonance frequency and equal volumes (A, B) or equal maximum transversal widths (C, D) with respect to the reference rectangular shape.

As shown above, the performance of the scavengers will depend on the applied resistive load. In Fig. 7 are thus shown the scavenged specific powers per unit of piezoelectric material volume as function of the connected resistances obtained by employing Roundy's analytical model. The diagrams correspond to the layouts given in Fig. 6. It can be seen that each configuration has a maximal power generation for a given value of resistive load. In this regard the efficiency of prototypes C and D seems better for a wider range of resistive loads.

### 3.2 FEM modeling of optimized layouts

The above described analytical models available in literature<sup>[11, 12]</sup>, although helpful in defining relevant parameters for the harvested power levels, are significantly simplified from the structural point of view. In order to obtain a more detailed description of the electromechanical behavior of the system, involving effects such as stress concentration and charge distribution, accurate analyses, as for example by means of finite elements models (FEM), should be performed.

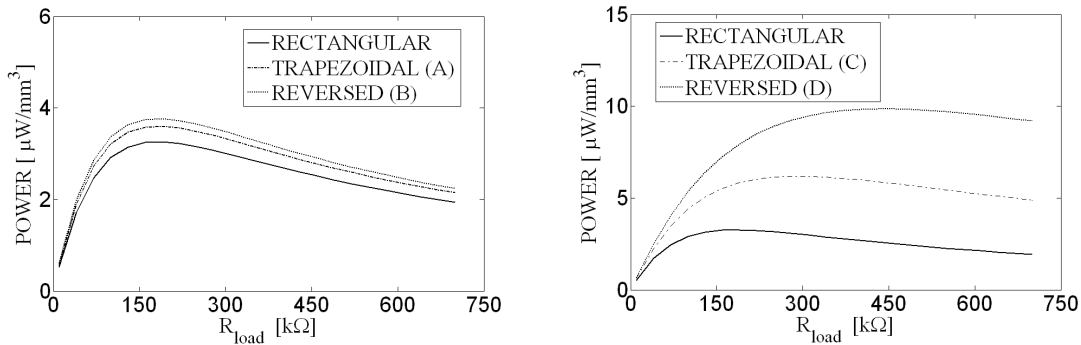


Fig. 7. Performances of the scavenger structures poled for series operation.

A FEM model is hence developed by means of the commercial ANSYS code. A 3D model is built with SOLID45 elements for the metallic structure and the inertial mass and SOLID5 elements for the two piezoelectric layers, for a total of 22780 electric and mechanical degrees of freedom (DOFs). In a first step, a modal analysis of all optimized scavenger shape configurations is performed to calculate their natural frequencies and eigenmodes. As an example, in Fig. 8 are shown the first four modes of vibration of the scavenger with rectangular shape (respectively at 50, 250, 320 and 1900 Hz) and of that with trapezoidal shape (50, 200, 235 and 2300 Hz). It is worth noting here that the sequence of computed eigenmodes in the frequency domain includes a second torsional mode. However, resonance frequencies are well separated and external actions can thus be clearly applied to the first eigenmode without exciting torsional vibrations.

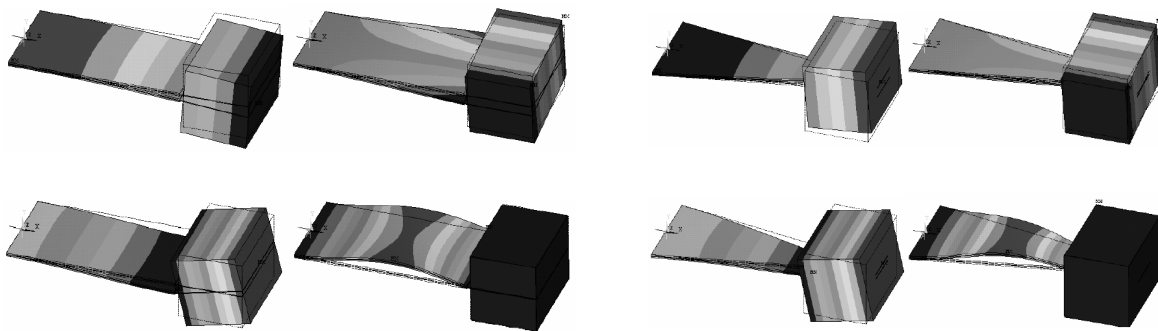


Fig. 8. First four vibration modes of the rectangular and trapezoidal scavengers.

To calculate the power generated on the studied geometries, a forced dynamic (harmonic) analysis is implemented next. Excitation of sinusoidal form is applied at the cantilever fixed end assuming constant excitation acceleration while maintaining the excitation frequency equal to the resonance frequency. The displacements along the cantilever are computed allowing, by employing coupled electromechanical analysis, the stress and the charge distributions on the piezoelectric layers to be identified. This, in turn, allows the voltage and scavenged power estimation.

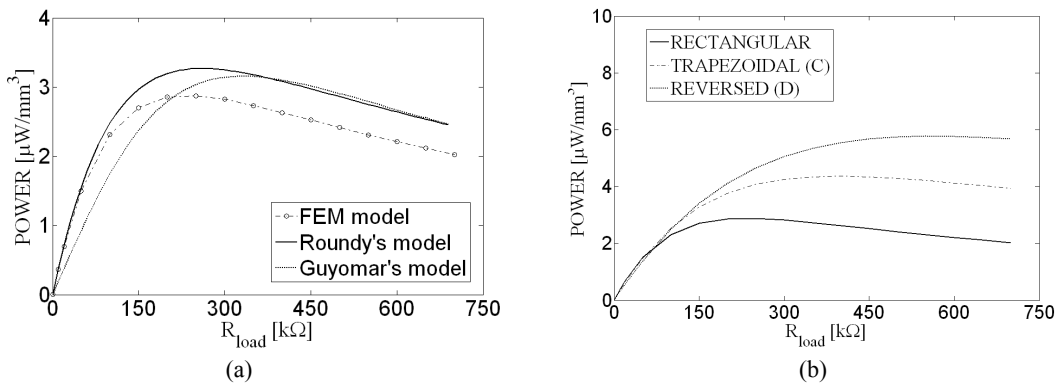


Fig. 9. FEM and analytical variations of the average power on the rectangular scavenger for variable resistive loads (a) and FEM results for the scavenger shapes of equal maximum transversal widths (b).



Using the chosen FEM code it is possible to perform also the computation of the variation of the voltage and power distribution by varying the applied resistive loads. For this purpose the nodes of the upper and lower surfaces of the FEM mesh are connected via CIRCU94 elements having the properties of a resistance. The resulting power values vs. the resistive load on the rectangular scavenger are shown in Fig. 9a, where the marked similarity, but also the differences, due to the evidenced limits of Roundy's and Guyomar's analytical models, can be appreciated. The approximations of the analytical models seem to result in an overestimate of the calculated specific power levels per unit volume of the piezoelectric material. In the case of Guyomar's model, the displacements  $u_m$  used in eq. (13) are those calculated via FEM harmonic analysis. The results for the scavenger shapes of equal maximal widths calculated by employing FEM are shown in Fig. 9b and have the same trends of those given in Fig. 7.

### 3.3 Obtained results

In Fig. 10 are shown the obtained results, in terms of voltage distributions along the considered scavenger shapes, resulting from FEM analyses. It can be observed that these follow the stress distributions given in Fig. 5. A smoother voltage distribution on the surface of the piezoelectric layers is obtained in the case of trapezoidal shapes. However, the efficiency of trapezoidal shapes is strongly affected by the clamp, since the width of this shape is large at the fixed end. This effect, which cannot be appreciated by using the analytical models, affects also the mode shape of the trapezoidal scavengers. All of this causes a lowering of the obtainable power levels that can be appreciated also by comparing Fig. 9b and Fig. 7. and could indicate that it might be better not to clamp the piezoelectric layers but only the metallic shim. In the case of the reversed trapezoidal configurations, a charge peak, corresponding to the highest stresses at the fixation of the cantilever, is obtained. In this region the width of the scavenger is small but, since the contribution of the peak on the average voltage is significant, clamping has a significant effect on the average achievable powers for the reversed trapezoids as well.

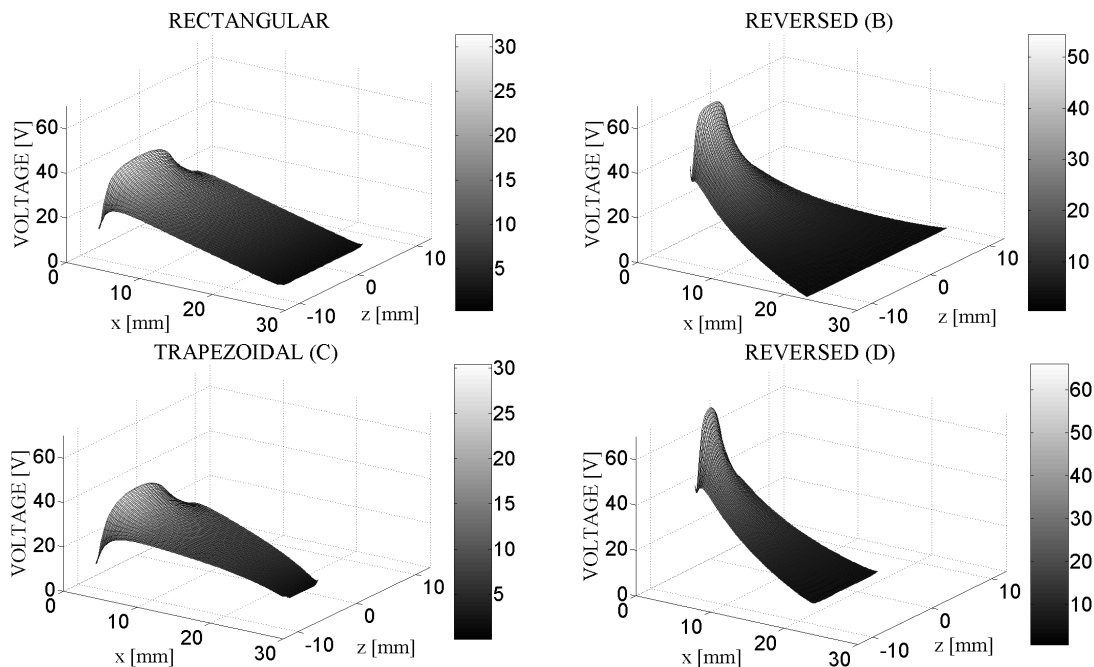


Fig. 10. Analysis of voltage distributions at the surface of the considered energy scavengers.

Although the capacitance of the transducer depends on its surface, and thus the shapes having equal volume ought to allow obtaining similar electric capacitances, due to the marked difference in charge distribution, this is not the case. This reflection is qualitative in nature, since the surface electrode covers the whole piezoelectric layer so that the presence of high conductivity distributes immediately the charge. Given the considered frequencies, the phenomenon of charge entrapment, dependant on the transient operation of charging and discharging of the capacitor<sup>[20]</sup>, is thus negligible. Nevertheless, it is evident that the voltage distributions and average values shown in Fig. 10 can be used to compare the power generated by the studied devices as well as that they differ sufficiently to give the designers additional parameters to refine the performed optimization. In Table 3 are shown the relevant values for the comparison of the performances of the proposed geometries. The assumptions of the analytical model proposed by Roundy affect the

computed values of the frequency corresponding to the first eigenmode of vibrations. However, all the prototypes are tuned on the same resonance. The maximal scavenged specific powers per unit of volume are at about  $6 \mu\text{W}/\text{mm}^3$ .

Table 3. Results of the analyses on the proposed geometries.

	Rectangular	EQUAL VOLUME		EQUAL MAX. WIDTHS	
		Trap. (A)	Rev. trap. (B)	Trap. (C)	Rev. trap. (D)
Freq. Roundy [Hz]	52.5	52.4	52.5	52.5	52.6
Freq. FEM [Hz]	49.3	46.8	48.9	46.1	48.1
Max. power FEM [ $\mu\text{W}/\text{mm}^3$ ]	2.9	2.9 (+0%)	2.4 (-17.2%)	4.4 (+51.7%)	5.8 (+100%)
Average voltage FEM [V]	24.36	24.44 (+0.3%)	20.95 (-14%)	30.39 (+24.7%)	33 (+35.5%)

FEM analyses confirm thus a significant increase of energy scavenging in case of the optimized geometric configurations of equal maximal widths with respect to the conventional rectangular shape. In this case the limitations due to the clamping effect are thus largely compensated by a better usage of the material, i.e. higher average strain levels. The highest improvement (up to 100%!) in specific power per unit of piezoelectric material volume is obtained in the case of the reversed trapezoidal shape, although a very marked advantage is obtained also by using the trapezoidal shape of equal maximal width. The equal volume criterion, contrary to what the analytical models would predict, results in a lowering of the obtained specific powers mostly due to the effects of the cantilever fixation visible in Fig. 10.

#### 4. EXPERIMENTAL VALIDATION

A preliminary experimental validation of the numerical model developed in this study is set-up. Specimens are made of two PSI-5A4E 0.2 mm thick PZT layers surface bounded on a 0.1 mm thick stainless steel shim (Fig. 11a) and poled for series operation<sup>[19]</sup>. The samples loaded with a suitable proof mass are excited via a PM25 MB Dynamics shaker (Fig. 11b). Excitation can be set at a given frequency imposing concurrently a constant acceleration or a constant displacement. For these purposes the shaker is driven by a 2635 Brüel & Kjaer amplifier and a 1047 Brüel & Kjaer exciter controller. Excitation acceleration is measured by 4375 Brüel & Kjaer accelerometers, while the resistive loads are adjusted via potentiometers. Displacements of the scavenger tip are monitored via a Micro-Epsilon OptoNCDT 1605 laser measurement system. All the data, including the obtained voltages, are collected via a PXI-1042 National Instruments data acquisition system and elaborated by employing LabVIEW-based algorithms.

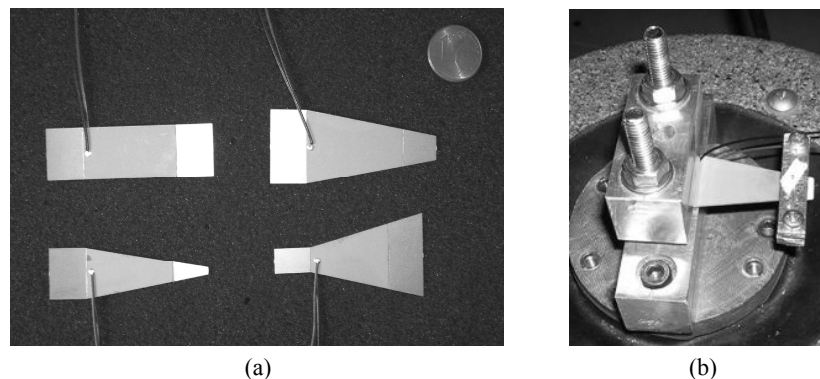


Fig. 11: Specimens of optimized energy scavenger shapes (a) and experimental set-up (b).

The electric characteristics of the piezoelectric transducers are measured employing a Hewlett-Packard 4284 capacitance measurement system. By measuring the capacitance and resistance (i.e. impedance) for variable excitation frequencies, it is verified that the transducers behave as a pure capacitor (i.e. that the resistive component of the transducer behavior is negligible)<sup>[21, 22]</sup>, while, as shown in Fig. 12a, their impedance is inversely proportional to excitation frequency<sup>[21, 22]</sup>.

By measuring the acceleration of the specimen fixation via the accelerometer and concurrently the displacement of its free end via the laser system, the typical transmissibility curves, such as depicted in Fig. 12b, are obtained. These allow determining the mechanical damping ratio  $\zeta$  of the studied structures, which is thus determined to be about 0,8%.

Fixing the excitation frequency to be equal to the first resonance frequency, the excitation acceleration is fixed, while the resistive load  $R_{load}$  is varied from  $1 \Omega$  to  $660 \text{ k}\Omega$ . The values of the output voltage are measured concurrently. It is to be

noted here that Young's modulus of the piezoelectric material, and thus its dynamic response, vary, although moderately, for changing resistive loads<sup>[22]</sup>. Measuring the open and short circuit response of the system, the electromechanical coupling coefficient  $k_{31}$  can hence be experimentally determined<sup>[11]</sup>. In this case it has the value of  $k_{31} = 0.13$ .

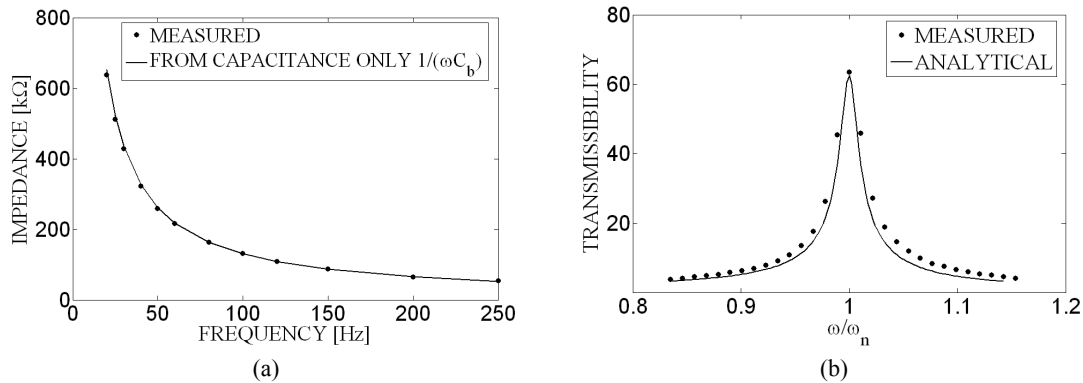


Fig. 12: Impedance (a) and transmissibility (b) of the specimens of optimized energy scavengers.

Preliminary results in terms of the obtained specific average output powers from the considered scavenger configurations at low accelerations are reported in Fig. 13. It can be clearly seen that the experimental curves have the same trends as the theoretical ones and that the trapezoidal shape produces significantly higher powers than the rectangular one. The difference in the values of the resistive load at which maximum power is obtained is consistent with the difference in the respective capacitances. The obtained powers vary almost linearly with the excitation acceleration.

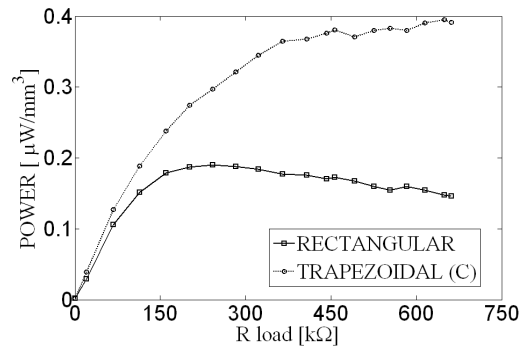


Fig. 13: Specific average output powers obtained on the studied scavenger shapes.

The experimental set-up is still in development and thus the systematic validation of the studied specimens is ongoing. The problems to be still dealt with include a more elaborated control of the clamping at the fixed end of the cantilever. This includes also a validation of the possibility to clamp the piezoelectric layer itself, as supposed in Roundy's model<sup>[11]</sup>, which would enable the comparison of the obtained results with this model but enhances the problems outlined in Fig. 10. The fastening of the proof mass so as to guarantee reproducible resonance frequencies, to avoid eventual short circuits, as well as to inhibit eventual parasitic influences of secondary vibration modes, is also an issue. Last but not least, the delivery of specimens where the volume of the piezoelectric layers would be perfectly known and consistent with the optimized shapes is still under negotiation with the suppliers. Nonetheless, the analytical, numerical and experimental results obtained so far clearly show that, in terms of generated power, optimized piezoelectric energy scavenger shapes could lead to significantly better specific performances.

## 5. CONCLUSIONS AND OUTLOOK

The recent build-up of wireless sensor networks for environmental and elder care monitoring requires a suitable development of portable power generation and accumulation devices called energy scavengers. In this work the electromechanical optimization of the shapes of piezoelectric cantilever resonators able to convert environmental vibrations into electrical energy is performed. As an alternative to the conventional rectangular cantilever studied in literature, two trapezoidal configurations, having respectively the wider side clamped or free, are proposed. Analytical

models given in literature are applied to obtain a preliminary evaluation of the scavenged powers. Numerical models, developed by using the finite element method, and taking into account all the relevant structural and electromechanical coupling effects, are then implemented. It is thus shown that the proposed geometries with equal maximal widths, if compared to the reference rectangular shape, allow considerable gains in the achieved power levels. The reached specific powers per unit of volume seem compatible with the power requirements of currently used wireless sensors. A preliminary experimental campaign on energy scavenger prototypes demonstrated the validity of the developed numerical model as well as the better performances of the optimized trapezoidal scavenger shapes. A thorough experimental analysis aimed at solving some of the encountered practical difficulties is currently being carried on. The work will proceed by considering the possibility of including passive and active tuning of the resonances of optimized devices to assure the maximal energy scavenging regardless of the uncertainties in the design and fabrication parameters, in the constraint conditions and in the excitation frequencies. The analyses will certainly have to include also material strength limits and the charge entrapment phenomenon. The latter could imply that the uniform stress criterion might not be sufficient to improve further the scavenged power levels without a concurrent electric capacitance optimization.

## REFERENCES

- [1] Helal, A., Mokhtari, M. and Abdulrazak, B., [The Engineering Handbook of Smart Technology for Aging, Disability and Independence], Wiley (2008).
- [2] Priya, S. and Inman, D. (ed.), [Energy Harvesting Technologies], Springer, New York (2009).
- [3] Starner, T. and Paradiso, J. A., "Human Generated Power for Mobile Electronics," in [Low Power Electronics Design] – ed. Piguat G., CRC Press, Boca Raton, ch. 45 (2004).
- [4] Anton, S. R. and Sodano, H. A., "A review of power harvesting using piezoelectric materials (2003 – 2006)," Smart Mater. Struc. 16, R1-R21 (2007).
- [5] Paradiso, J. A. and Starner, T., "Energy Scavenging for Mobile and Wireless Electronics," IEEE Perv. Comp. 4(1), 18-27 (2005).
- [6] Tech-UP – Engineering Laboratory for Ubiquous and Pervasive Technologies, Regional Scientific and Applied Research Project and Initiative for Transfer and Dissemination of Research Results, D. P. Reg. n. 0324/Pres. (2004).
- [7] Beeby, S. P., Tudor, M. J. and White, N. M., "Energy harvesting vibration sources for microsystems applications," Meas. Sci. Techn. 17, R175-R195 (2006).
- [8] Renno, J. M., Daqaq, M. F. and Inman, D. J., "On the optimal energy harvesting for a vibration source," J. Sound Vibr. 320, 386-405 (2009).
- [9] Carson, J., [Springs design handbook], Dekker (1978).
- [10] Genta, G., [Vibrations of structures and machines], Springer (1998).
- [11] Roundy, S. and Wright, P. K., "A piezoelectric vibration based generator for wireless electronics," Smart Mat. & Struct. 13, 1131-1142 (2004).
- [12] Guyomar, D., Badel, A., Lefeuvre, E. and Richard, C., "Toward Energy Harvesting Using Active Materials and Conversion Improvement by Nonlinear Processing," IEEE Trans. Ultras., Ferr. & Freq. Cont. 52(4), 584-595 (2005).
- [13] Roundy, S. et al., "Improving Power Output for Vibration-Based Energy Scavengers," IEEE Perv. Comp. 4(1), 28-36 (2005).
- [14] Shahruz, S. M., "Limits of performance of mechanical band-pass filters used in energy scavenging," J. Sound & Vibr. 293, 449-461 (2006).
- [15] Brusa, E., Carabelli, S., Carraro, F. and Tonoli, A., "Electromechanical Tuning of Self-Sensing Piezoelectric Transducers," J. Intel. Mat. Sys. & Struct. 9, 198-209 (1998).
- [16] Crandall, S. H. et al., [Dynamics of mechanical and electromechanical systems], McGraw Hill (1968).
- [17] Crawley, E. and Hall, S., [The dynamics of controlled structures], Technical Course, MIT, Boston, vol. 1-3 (1991).
- [18] Meirovitch, L., [Dynamics and Control of Structures], Wiley (1990).
- [19] Piezo Systems Inc., [www.piezo.com](http://www.piezo.com)
- [20] Bonse, M., [Capacitive position transducers theoretical aspects and practical applications], PhD, U. Deft (1995).
- [21] Jordan, T. L. and Ounaies, Z., "Piezoelectric ceramics characterization," NASA/CR-2001-211225, ICASE Report No. 2001-28 (2001).
- [22] Law, H. H., Rossiter, P. L., Simon, G. P. and Koss, L. L., "Characterization of mechanical vibration damping by piezoelectric materials," J. Sound & Vibr. 197(4), 378-402 (1996).